Horizon Academy Trust

Where anything is possible

EAST PARK PRIMARY SCHOOL

Calculation Policy

HORIZON VISION

Horizon recognises that every school is unique and that it serves its own unique community. Each school within Horizon maintains its own sense of identity, as we believe that they understand how to best serve their community.
We believe that in every academy within the trust, outstanding practice exists, which we utilise fully to underpin continued success and sustainability for all.
We have one simple tagline to describe our Trust, which we believe encompasses what we try to instil in our children: ***'Where anything is possible.***

Cavendish Primary School has adopted a Mastery style of teaching mathematics therefore within lessons pupils should experience the following elements within their daily mathematics lesson.

- A coherent Learning Journey which includes the following elements:

Concrete – Pictorial – Abstract – which enables Fluency - Reasoning and Problem Solving

All four ***mathematical operations*** are separated into 3 different representations: **concrete, pictorial and abstract.**

Concrete representations are where physical equipment (or "manipulatives") are used to help understand a mathematical concept. Some examples are included in this document.

It is important that children experience a range of different representations when learning mathematical concepts. **Pictorial representations** are where drawings/pictures are used to help understand a mathematical concept. These can be just picture of a concrete representations (e.g., a drawing of place value counters) or it could be strategies such as bar models and part-part-whole models. Again, it is important that children experience a range of different representations.

Abstract representations are where numbers (or letters) are used to represent a mathematical concept. This is the representation which is typical thought of as "maths" and is usually the most efficient; however, understanding just an abstract representation (and not pictorial and concrete versions) can suggest that the actual mathematical concept is not completely understood.

At East Park Primary School, the learning journey incorporating the CPA approach is ***Experienced through:***
Guided Practice, Fluency and Reasoning and Problem Solving. Individual lessons will incorporate the following elements:

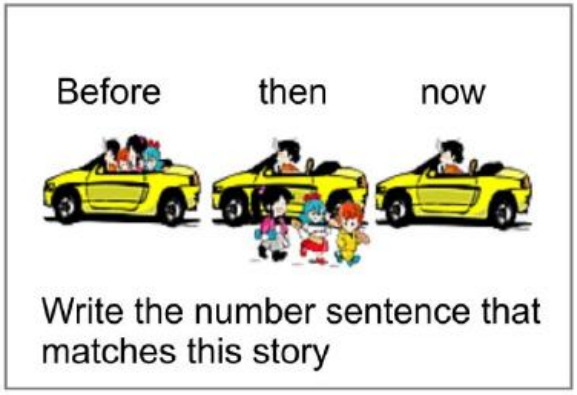
- A Range of representations should be used for a given concept.
- There should be examples of conceptual and procedural elements.
- Within calculating children will experience the following elements:

  That the = sign is a sign of equivalence

  Inequality should be taught alongside equality (< and >)



- Look for patterns and connections (what is the same / different)
- Use intelligent practice, for example if 2 x 3 = 6 then 20 x 3 = 60
- Use empty box questions, for example 20 + = 30
- Work systematically
- Use Before then now statements



- Contextualise the calculation within a problem.

Questioning which focuses on pupils explaining how they calculated in order to develop reasoning. For example:

- Which calculation is the odd one out?
- Identification of the correct answer / question for a given answer
- Which calculation is true / false?
- Which calculation is greater than / less than / equal to? How do you know?
- Correct use of Mathematical terminology.
- Use of sentence stems in order to express connections

POINT TO NOTE – THIS IS A WORKING DOCUMENT WHICH WILL BE REVIEWED AND UPDATED AS NECESSARY.

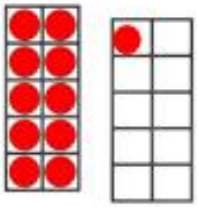
Examples of Concrete Resources which may be used within maths sessions



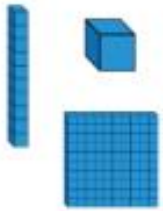
Numicon



Place Value Counters



Ten Frames



Base 10



Straw Bundles



Money





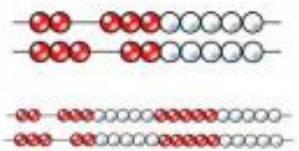
Weights



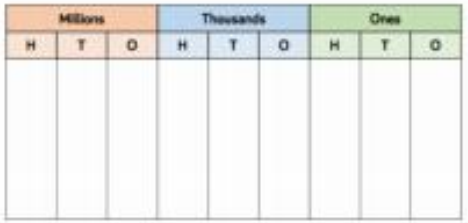
Cubes and counters



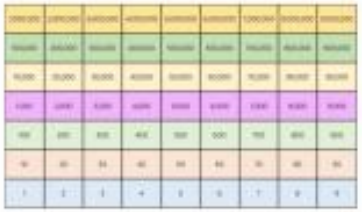
Arrow cards



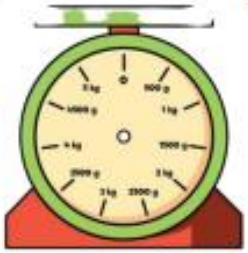
Bead strings



Place Value grid



Place Value chart



scales



Measuring cylinders



Cuisenaire rods

Examples of Vocabulary

Addition



- Sum
- Total
- Parts and wholes
- Plus
- Add
- Altogether
- More
- Is equal to
- Is the same as

Subtraction



- Take away
- Less than
- The difference
- Subtract
- Minus
- Fewer
- Decrease

Multiplication



- Double
- Arrays
- Multiplied by /multiply
- The product of
- Multiple of
- Groups of
- Lots of
- Equal groups
- Repeated addition
- Times

Division



- Share /share equally
- Group /equal groups
- Divide
- Divided by
- Divided into
- Half
- Remainder
- Quotient

Equals

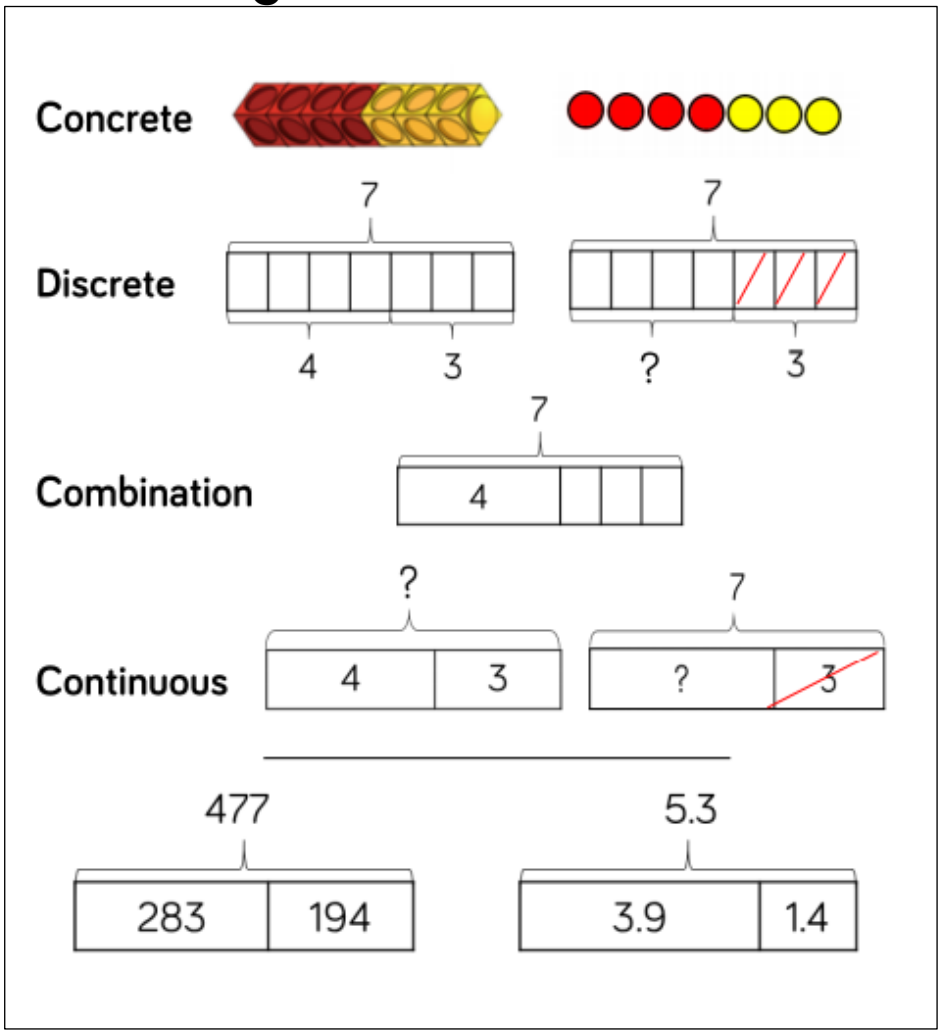


- Makes
- Total
- Same as
- Equivalent to
- Balances
- Equal

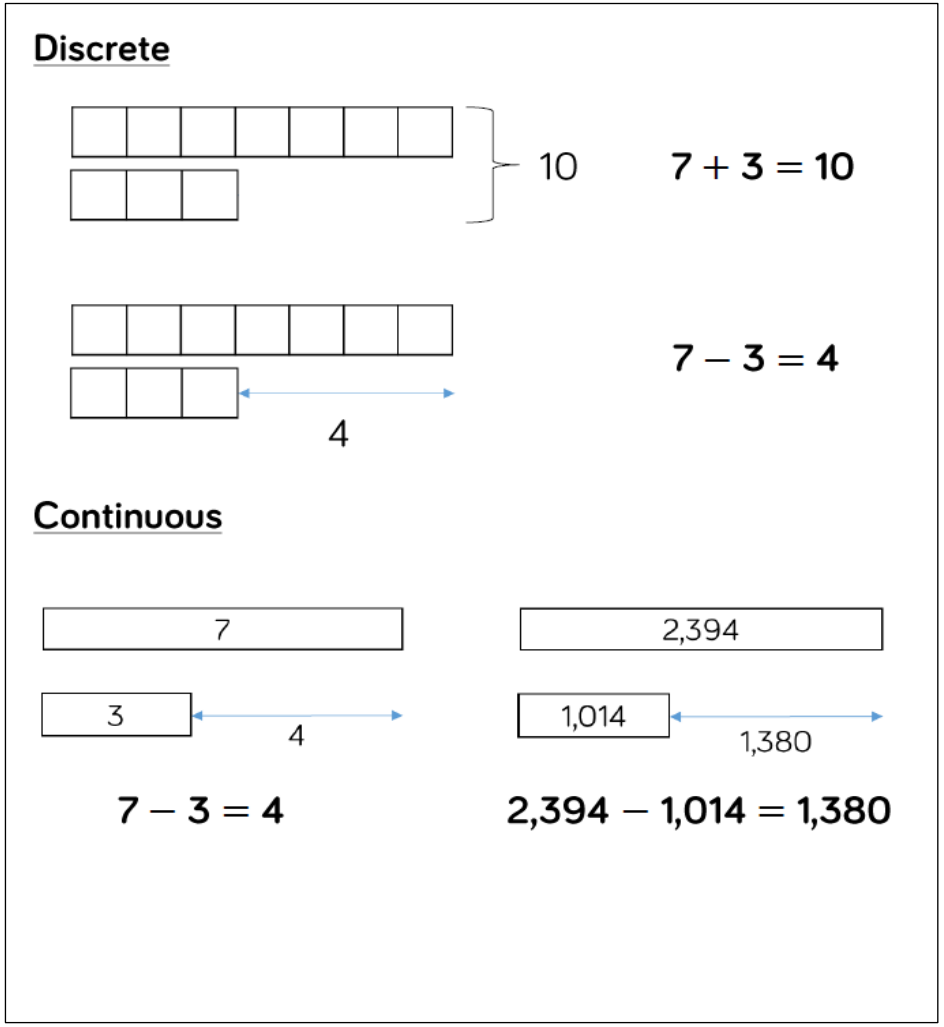
Representations and concrete resources which may be used for addition and subtraction:

Bar Models

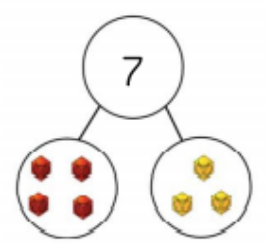
Single

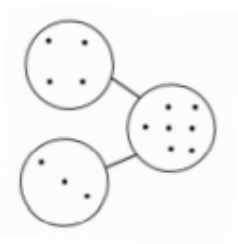


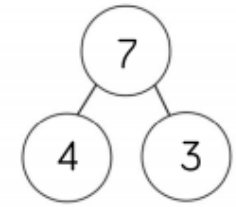
Double



Part Whole Models

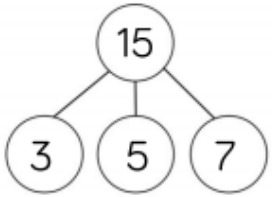


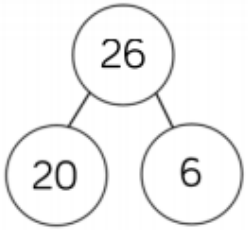


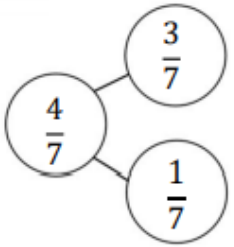


$7 = 4 + 3$
$7 = 3 + 4$

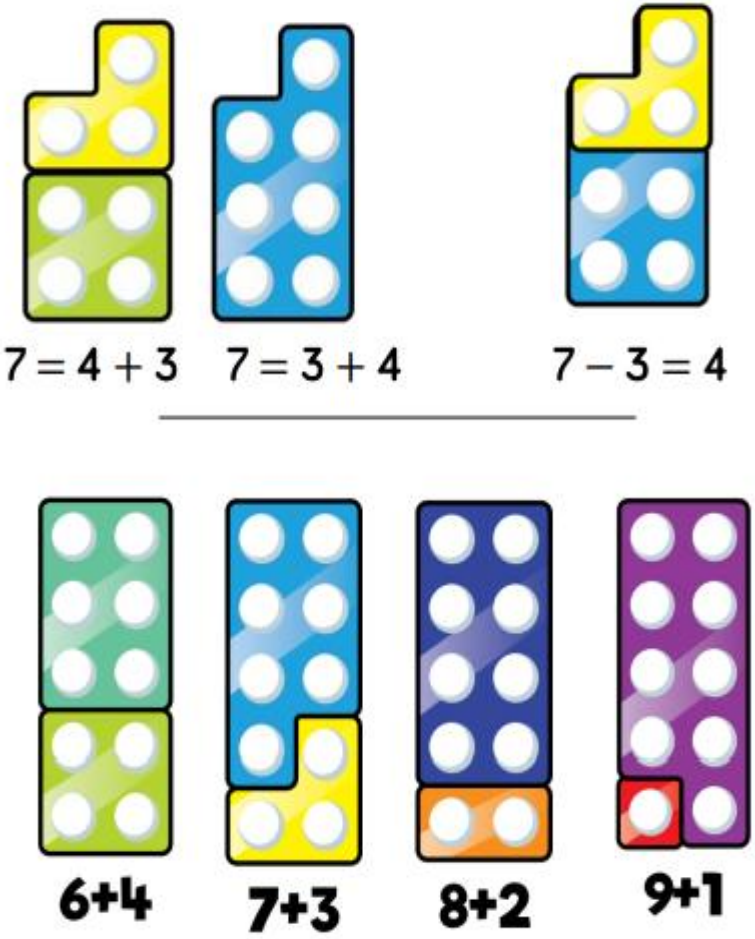
$7 - 3 = 4$
$7 - 4 = 3$



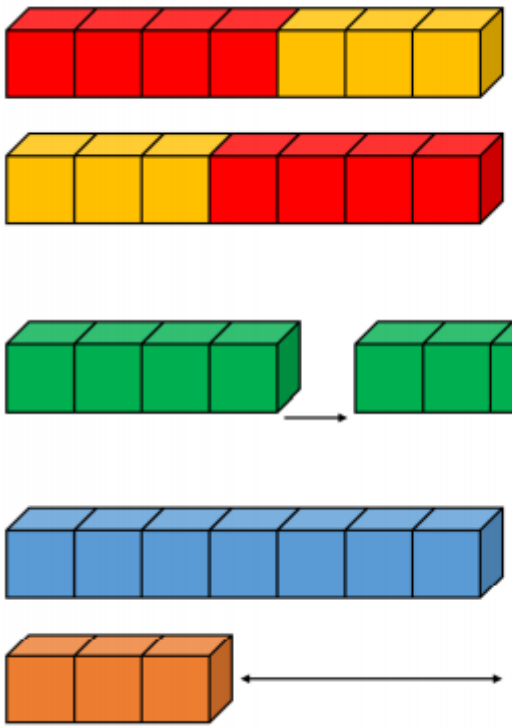




Number Shapes / Numicon



Cubes



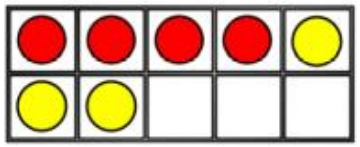
$7 = 4 + 3$

$7 = 3 + 4$

$7 - 3 = 4$

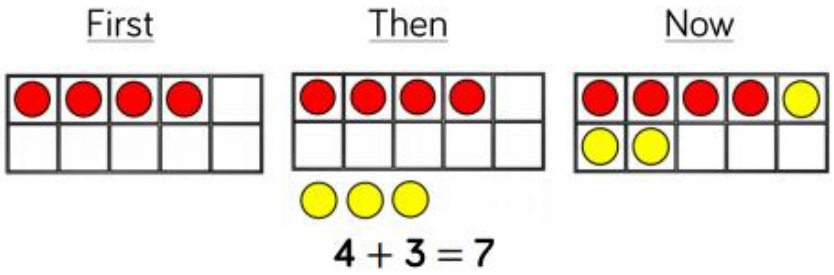
$7 - 3 = 4$

Tens Frames

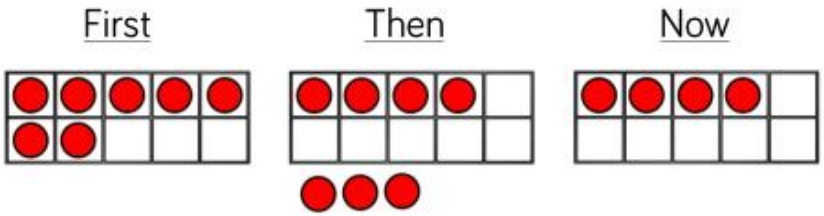


$4 + 3 = 7$
$3 + 4 = 7$
$7 - 3 = 4$
$7 - 4 = 3$

4 is a part.
3 is a part.
7 is the whole.

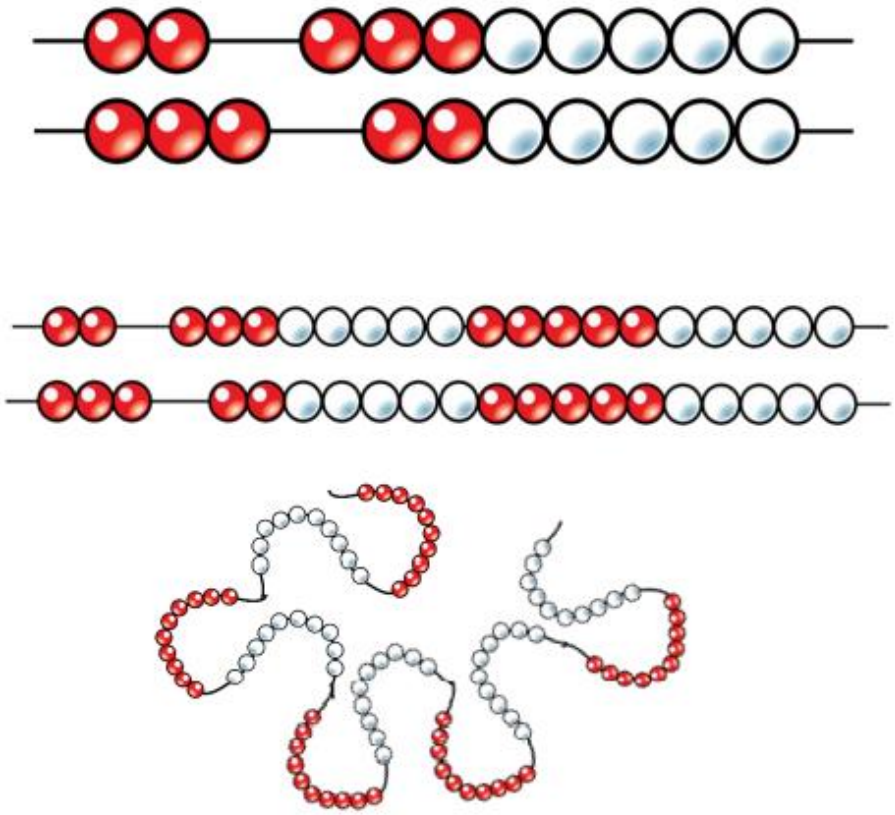


$4 + 3 = 7$



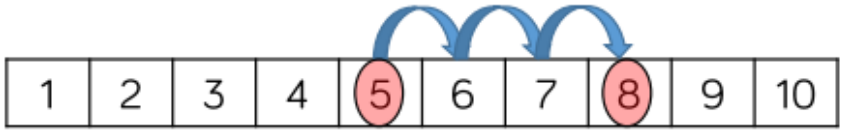
$7 - 3 = 4$

Bead Strings

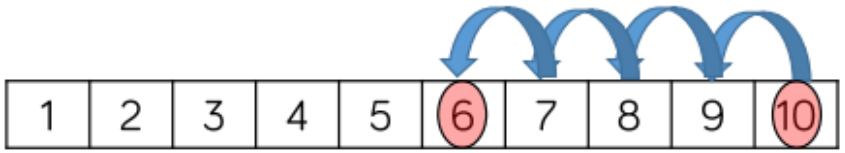


Number Tracks

$5 + 3 = 8$



$10 - 4 = 6$

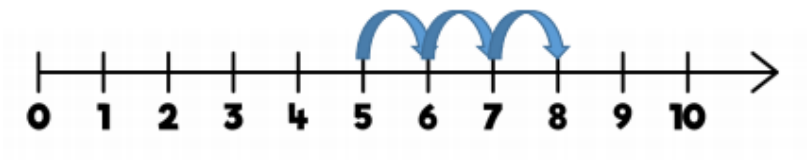


$8 + 7 = 15$

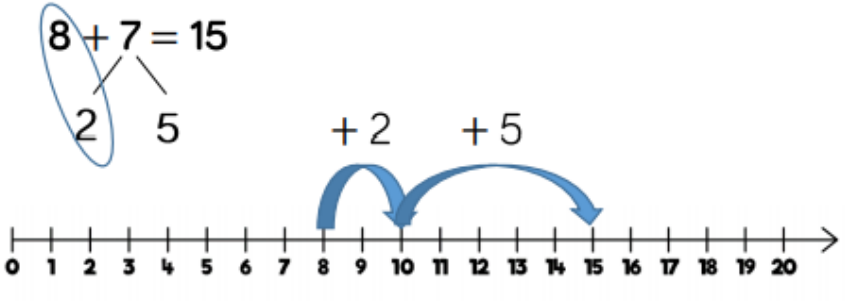


Labelled Number lines

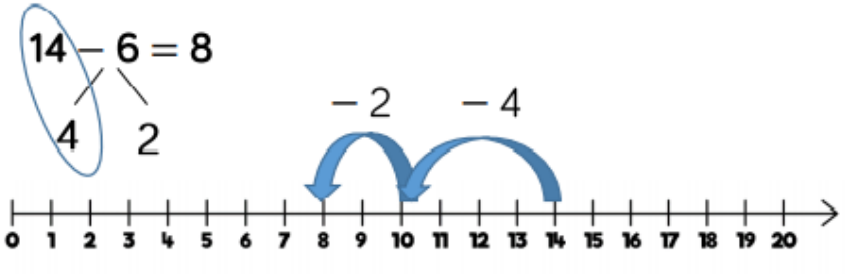
$5 + 3 = 8$



$8 + 7 = 15$

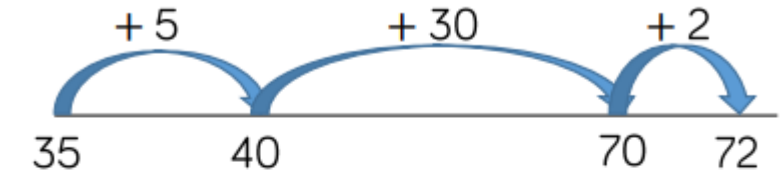


$14 - 6 = 8$

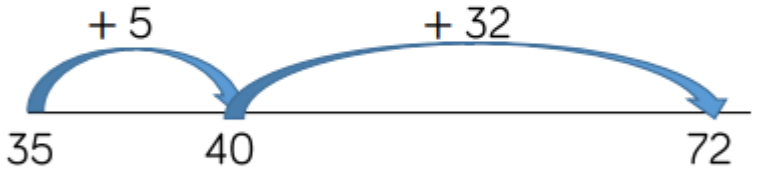


Blank Number lines

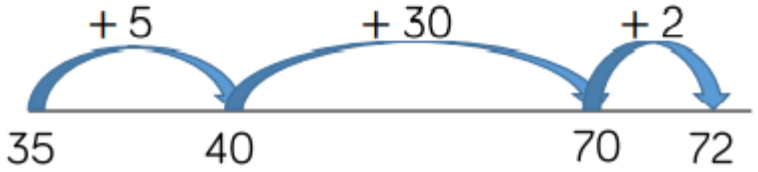
$35 + 37 = 72$



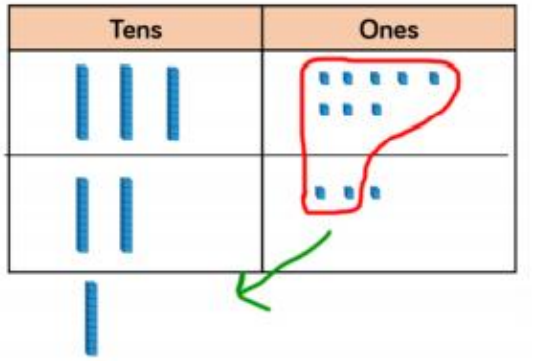
$35 + 37 = 72$



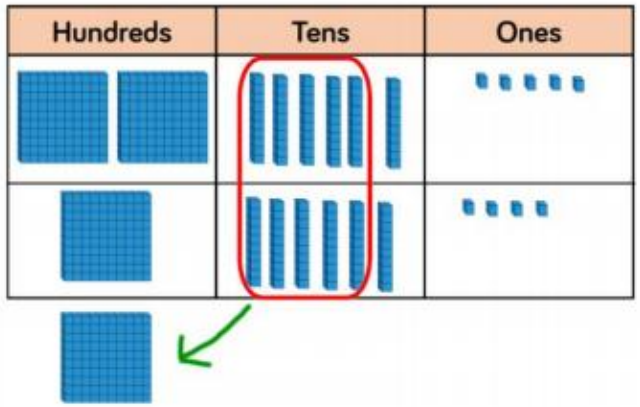
$72 - 35 = 37$



Base Ten (Dienes Apparatus)

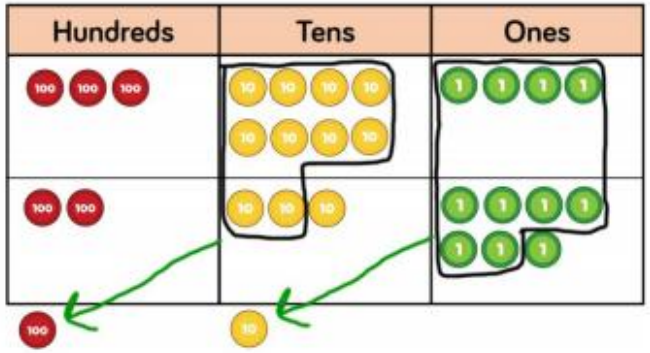


$$\begin{array}{r} 38 \\ +\ 23 \\ \hline 61 \\ \hline 1 \end{array}$$

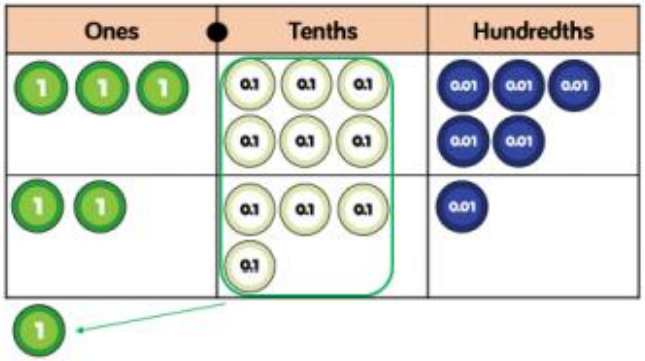


$$\begin{array}{r} 265 \\ +\ 164 \\ \hline 429 \\ \hline 1 \end{array}$$

Place Value Counters



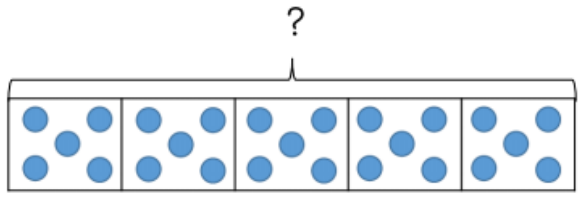
$$\begin{array}{r} 384 \\ +\ 237 \\ \hline 621 \\ \hline 1\ 1 \end{array}$$



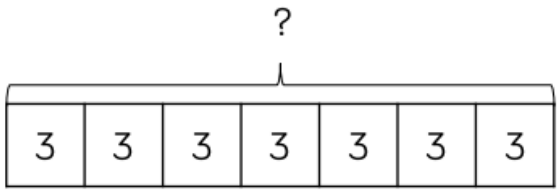
$$\begin{array}{r} 3.65 \\ +\ 2.41 \\ \hline 6.06 \\ \hline 1 \end{array}$$

Representations and concrete resources which may be used for multiplication and division:

Bar Model

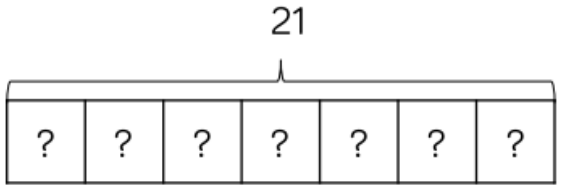


$5 \times 5 = 25$

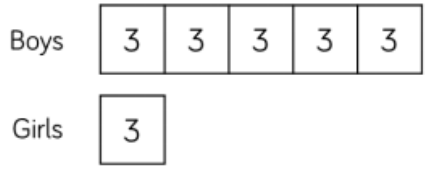


$3 \times 7 = 21$

$7 \times 3 = 21$



$21 \div 7 = 3$



Number Shapes (Numicon)



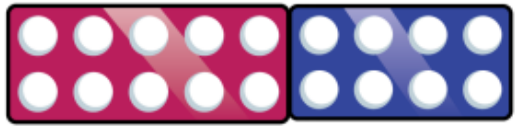
$5 \times 4 = 20$

$4 \times 5 = 20$

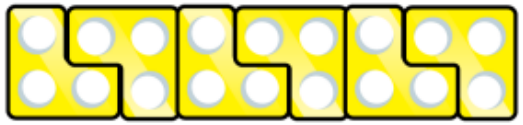


$5 \times 4 = 20$

$4 \times 5 = 20$



$18 \div 3 = 6$



Bead Strings



$5 \times 3 = 15$
$3 \times 5 = 15$

$15 \div 3 = 5$



$5 \times 3 = 15$
$3 \times 5 = 15$

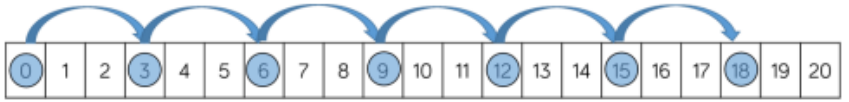
$15 \div 5 = 3$



$4 \times 5 = 20$
$5 \times 4 = 20$

$20 \div 4 = 5$

Number Tracks



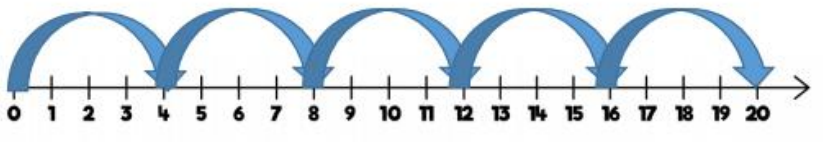


$6 \times 3 = 18$
$3 \times 6 = 18$



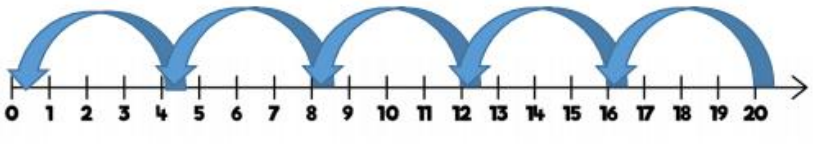
$18 \div 3 = 6$

Number Lines





$4 \times 5 = 20$
$5 \times 4 = 20$

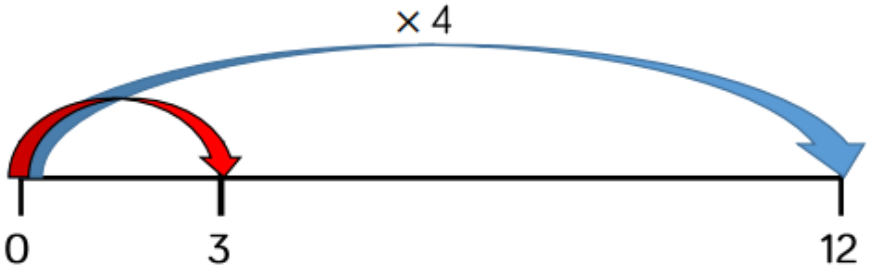


$20 \div 4 = 5$

Blank Number Lines



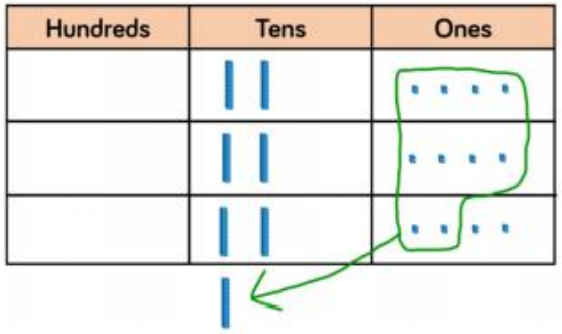
A red car travels 3 miles.
A blue car 4 times further.
How far does the blue car travel?



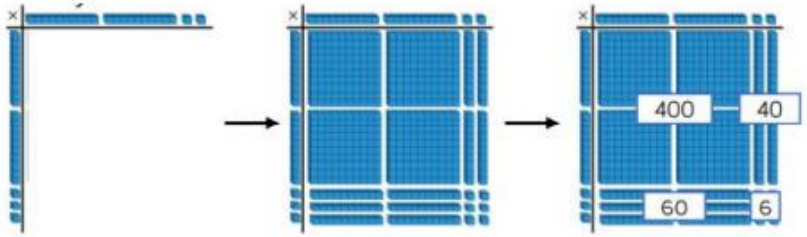
A blue car travels 12 miles.
A red car 4 times less.
How far does the red car travel?

Base Ten (Dienes)

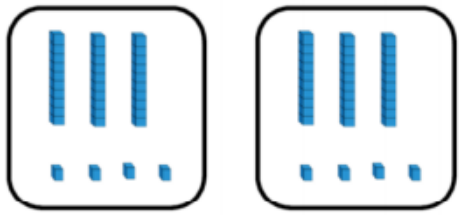
Multiplication



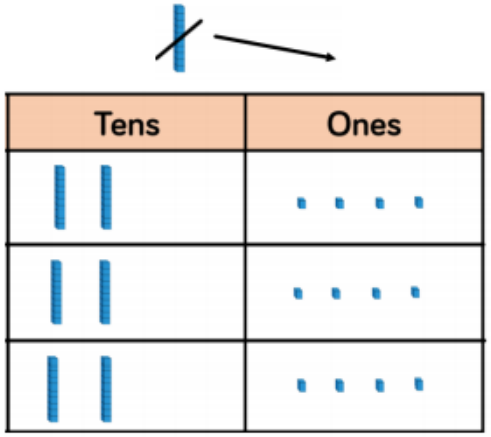
$$\begin{array}{r} 24 \\ \times\ 3 \\ \hline 72 \\ \hline 1 \end{array}$$



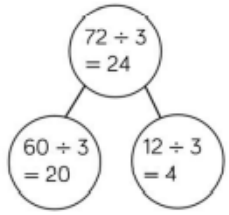
Division



$$68 \div 2 = 34$$



$$72 \div 3 = 24$$

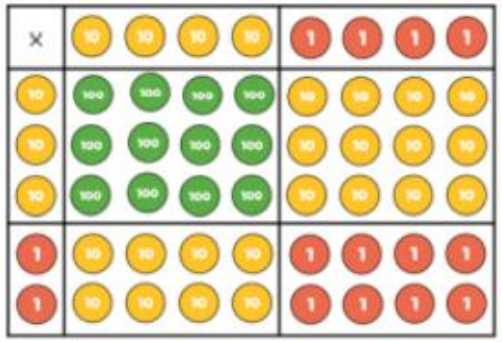


Place Value Counters

Multiplication

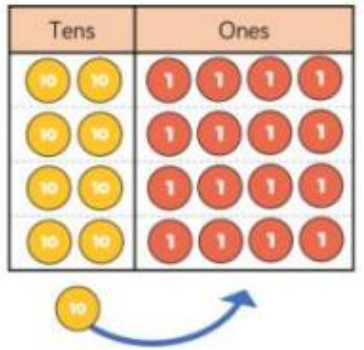


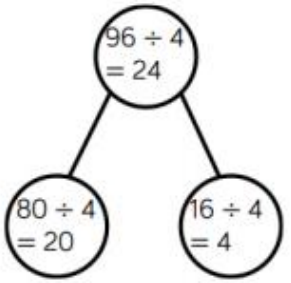
$$\begin{array}{r} 34 \\ \times\ 5 \\ \hline 120 \\ \hline 1\ 2 \end{array}$$

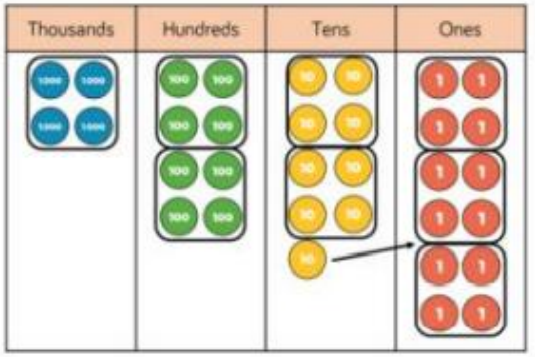


$$\begin{array}{r} 44 \\ \times\ 32 \\ \hline 8 \\ 80 \\ 120 \\ +\ 1200 \\ \hline 1408 \\ \hline 1 \end{array}$$

Division



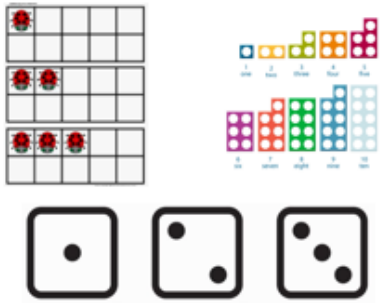
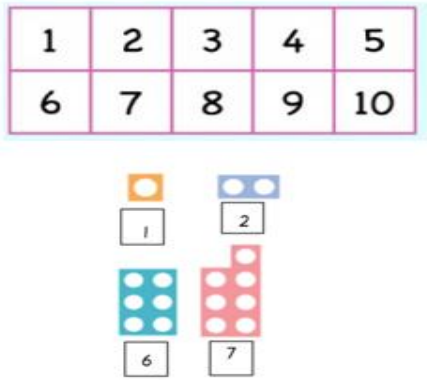




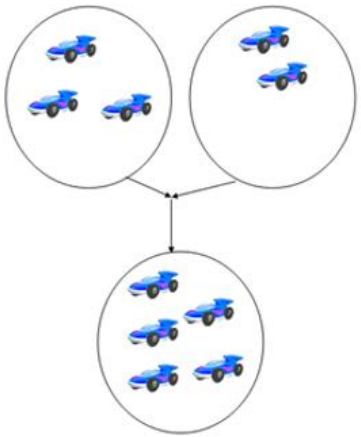
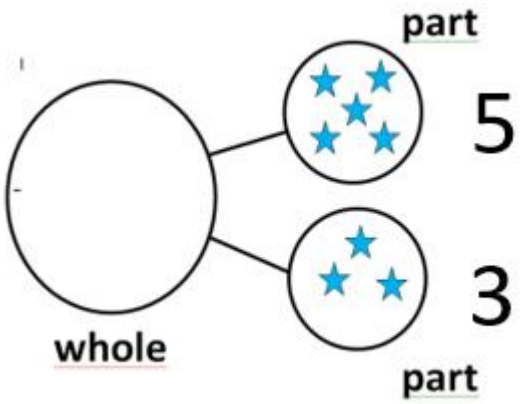
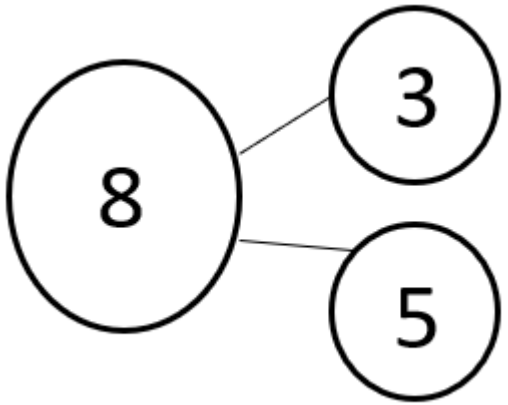
$$\begin{array}{r} 1223 \\ 4\overline{)\,4892^{1}} \end{array}$$

Addition

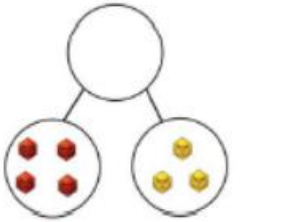
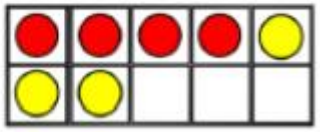
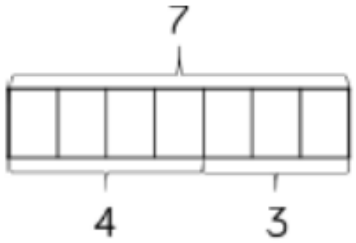
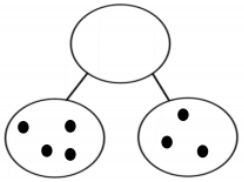
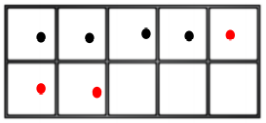
EYFS

Concrete	Pictorial	Abstract
Comparison of quantities  Children use toys/objects to compare which has more and fewer. Link to measures taller/shorter.	 Children have pictures of manipulatives to represent more or fewer. "I have 5. Show me fewer.	 Match numeral to amount.
Subitization  Children can manipulate objects e.g. ten frames (eggs in egg box, foam ten frames, ten frame trains), Numicon tiles, dice to show numbers 0-5 then 6-	 Children presented with image are able to use subitizing skills to instantaneously recognise the number	 Match numeral to amount.

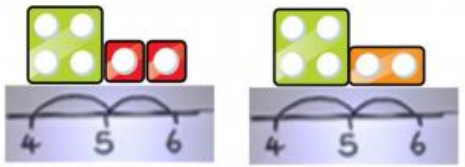
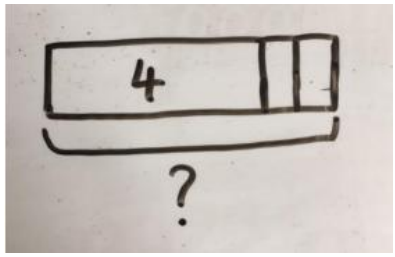
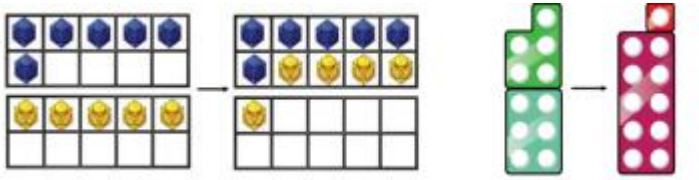
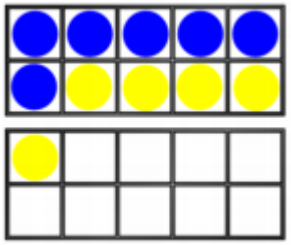
EYFS

Concrete	Pictorial	Abstract
Part and whole  Children make two sets of objects in a part, part, whole frame then combine it into one.	 Children use pictures to combine two amounts together as one whole. 3 is part, 5 is part and 8 is the whole.	 Numerals used to show the quantities in the parts and wholes alongside the pictorial e.g., could match numeral/digit cards to the amounts.

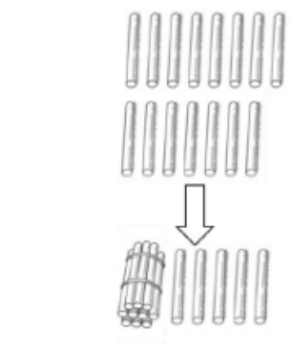
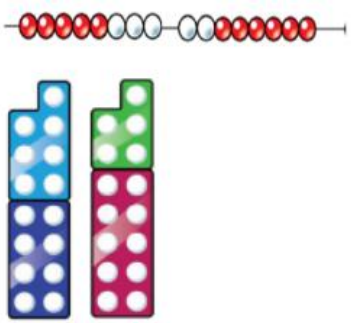
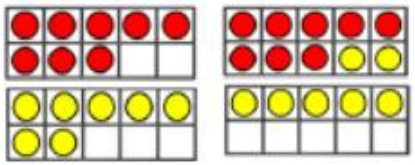
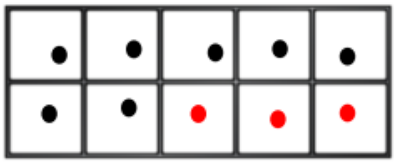
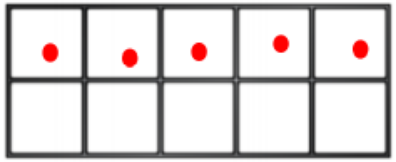
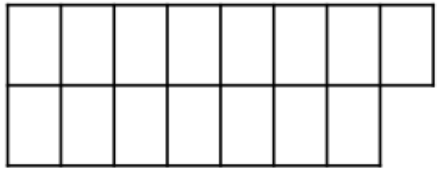
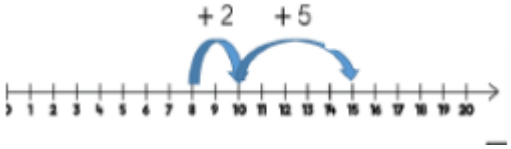
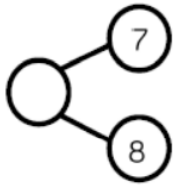
EYFS / YEAR 1 ADDITION WITHIN 10.

Concrete	Pictorial	Abstract
Addition within 10      	    	**4 + 3 = 7**

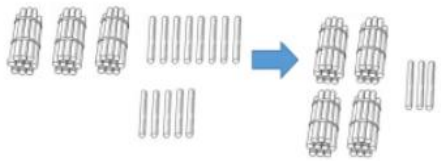
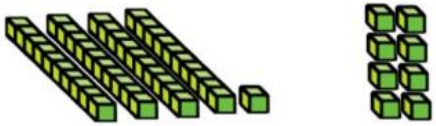
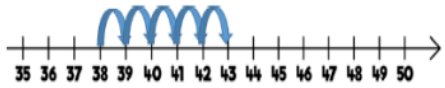
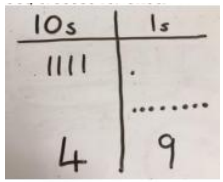
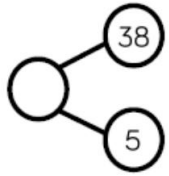
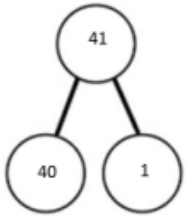
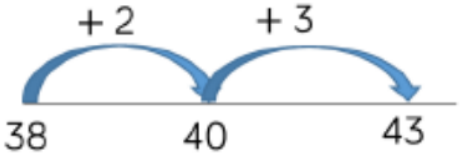
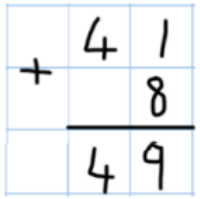
YEAR 1 – WORKING WITHIN 10

Concrete	Pictorial	Abstract
Counting on using number lines using cubes or Numicon.   	A bar model which encourages the children to count on, rather than count all.  	The abstract number line: What is 2 more than 4? What is the sum of 2 and 4? What is the total of 4 and 2? 4 + 2 
Regrouping to make 10, using ten frames and counters/cubes or using Numicon. 6 +5 **IMPORTANT AS IT IS THE FOUNDATION FOR COLUMN ADDITION LATER ON.** 	Children to draw the ten frame and counters/cubes. 	Children to develop an understanding of equality e.g. 6 + ▭ = 11 6 + 5 = 5 + ▭ 6 + 5 = ▭ + 4

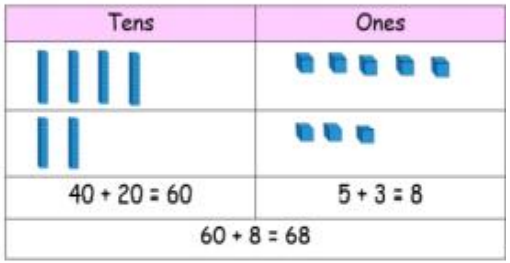
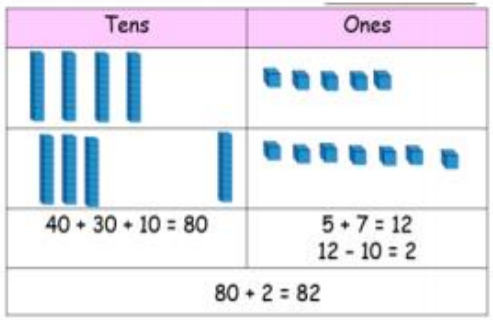
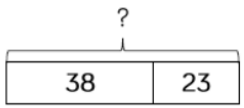
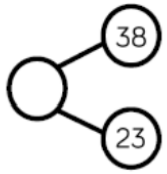
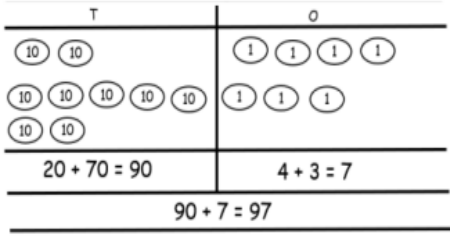
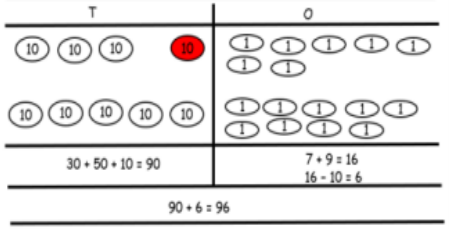
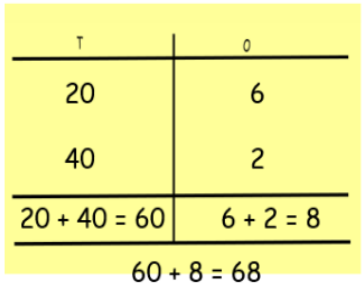
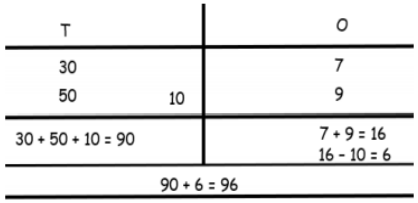
YEAR 1 – ADDITION WITHIN 20

Concrete	Pictorial	Abstract	Additional notes / guidance
  	    	**8 + 7 = 15** **8 + ___ = 15** **___ + 8 = 15**	❖ Draw a number line and put your starting number on. ❖ Make jumps to make the next ten. ❖ Add the rest of the ones.

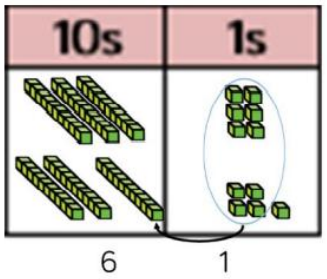
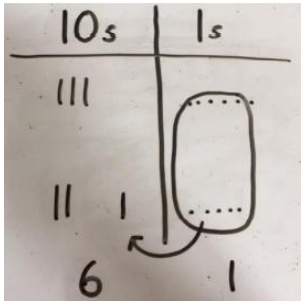
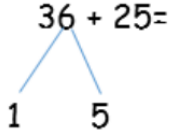
YEAR 2 - ADD 1 DIGIT AND 2-DIGIT NUMBERS WITHIN 100

Concrete	Pictorial	Abstract	Additional notes / guidance
 38 + 5  41 + 8 	 38 + 5   	 41 + 8 1 + 8 = 9 40 + 9 = 49 	❖ Draw a number line and put your starting number on ❖ Make jumps to make the next ten. Add the rest of the ones. ❖ Use lines and dots to represent the tens and ones in the pictorial method.

YEAR 2 – ADD TWO 2 DIGIT NUMBERS WITHIN 100

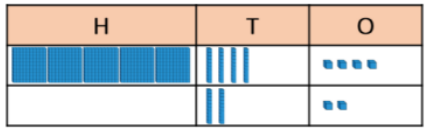
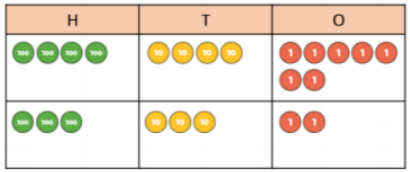
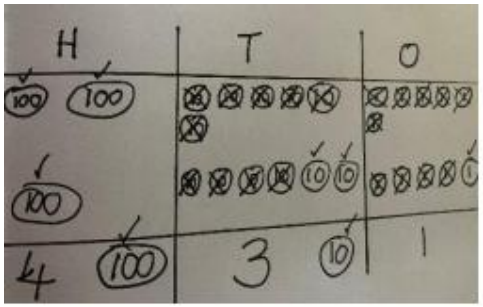
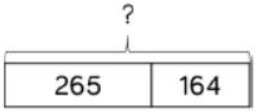
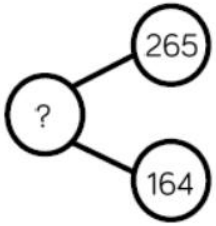
Concrete	Pictorial	Abstract	Additional notes / guidance
  	   	  **Formal method** 38 +23	❖ Place the ones into the place value chart ❖ Place the tens in the place value chart ❖ Find the total of the ones. Exchange 10 ones for 1 ten Find the total of the tens ❖ Add the total ones to the total tens.

YEAR 3 – ADD TWO 2 DIGIT NUMBERS WITHIN 100

Concrete	Pictorial	Abstract	Additional notes / guidance
TO + TO using base 10. Continue to develop understanding of partitioning and place value. 36 + 25  	Represent the base 10 in a place value chart.  	Looking for ways to make 10. 30 + 20 = 50 5 + 5 = 10 50 + 10 + 1 = 61  **Formal Method** 36 +25 61 1	❖ Ensure that columns are lined up when introducing the formal method.

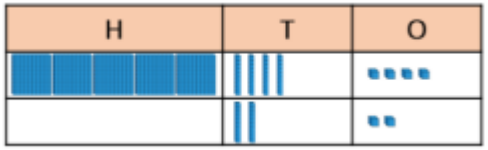
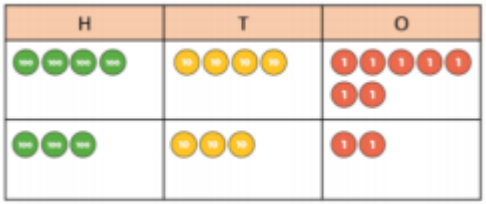
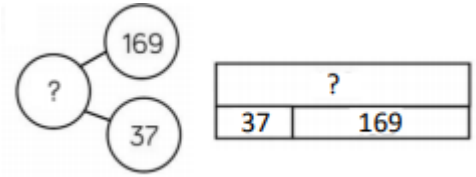
YEAR 3 – ADD NUMBERS WITH UP TO 3 DIGITS – COLUMN METHOD (without renaming / exchanging)

HTO + TO HTO + HTO

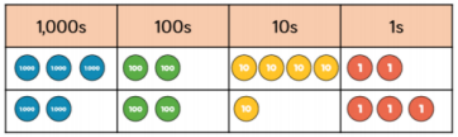
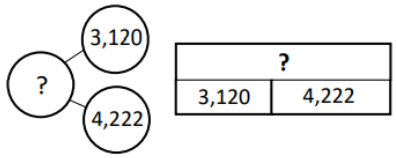
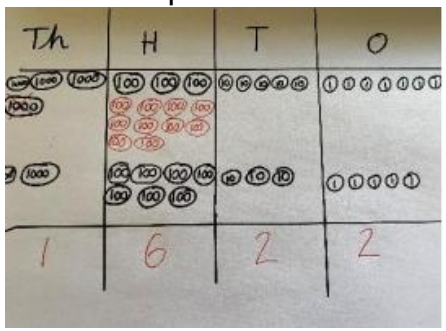
Concrete	Pictorial	Abstract	Additional notes / guidance
544 + 22 = 566  **447 + 332 = 779**  Children look at each column (starting with the ones) and count how many counters/dienes are in each.	Children to represent the counters in a place value chart.    	$\begin{array}{r} 265 \\ +\ 164 \\ \hline 429 \\ \hline 1 \end{array}$ 265 + 164 = 164 + 265 = Abstract column addition is taught alongside concrete and pictorial strategies to help link the concepts of column value - e.g. 5 tens + 2 tens = 7 tens	❖ Write column headings ❖ Write the number of ones, tens, and hundreds in the correct column. ❖ Starting from the ones column add the numbers in each column carrying to the next column if you need to.

YEAR 3- ADD NUMBERS WITH UP TO 3 DIGITS – COLUMN METHOD (with renaming / exchange)

TO + TO HTO + TO HTO + HTO

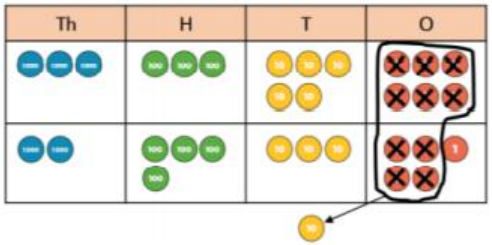
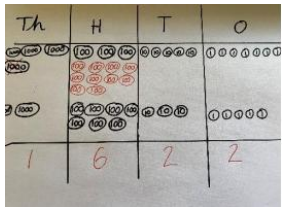
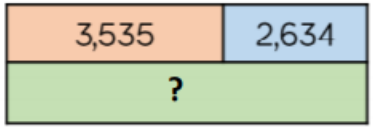
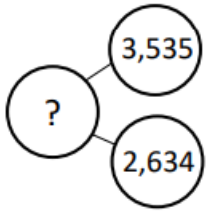
Concrete	Pictorial	Abstract	Additional notes / guidance
544 + 22 = 566  447 + 332 = 779  Children look at each column (starting with the ones) and count how many counters/dienes are in each.	Children to draw their own representations Pictures of Dienes and place value counters used as opposed to physical equipment. Part-part-whole and bar models are also used to help understand the concept of addition – particularly to help visualise word problems.  	$\begin{array}{r} 356 \\ +\ 523 \\ \hline 879 \end{array}$ Abstract column addition is taught alongside concrete and pictorial strategies to help link the concepts of column value – e.g. 5 tens + 2 tens = 7 tens	❖ Write column headings. ❖ Write the number of ones, tens, and hundreds in the correct column. ❖ Starting from the ones column add the numbers in each column exchanging from the next column if you need to.

YEAR 4 – ADD NUMBERS WITH UP TO 4 DIGITS (without exchange)

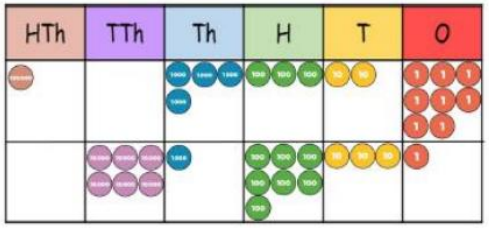
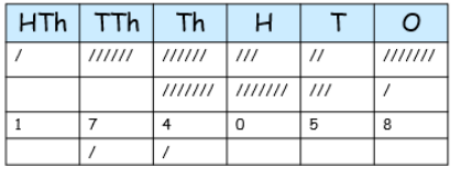
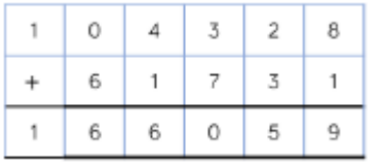
Concrete	Pictorial	Abstract	Additional notes / guidance
3,242 + 2,213 = 5,455  Children look at each column (starting with the ones) and count how many counters are in each. Dienes / base 10 could still be used to help with understanding of place value.	Part-part-whole and bar models are also used to help understand the concept of addition – particularly to help visualise word problems.  Children may also draw their own representations  	**3538** **+5251** **8789** Abstract column addition is taught alongside concrete and pictorial strategies to help link the concepts of column value - e.g. 5 hundreds + 2 hundreds = 7 hundreds	❖ Write column headings. ❖ Write the number of ones, tens, hundreds, and thousands in the correct column. ❖ Starting from the ones column add the numbers in each column exchanging from the next column if you need to.

YEAR 4 – ADD NUMBERS WITH UP TO 4 DIGITS (with exchange)

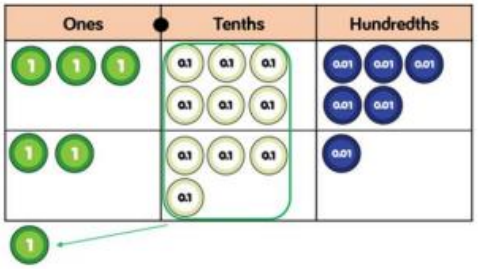
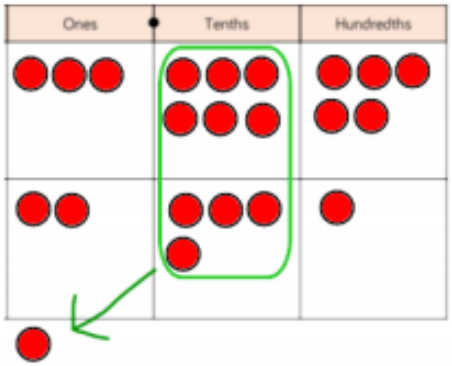
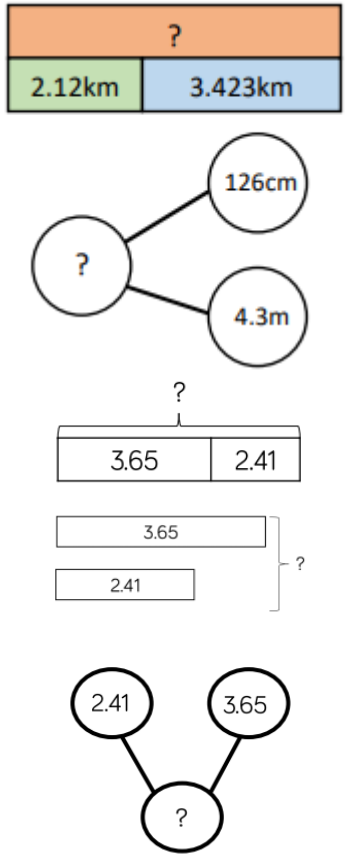
up to Th H T O + Th H T O

Concrete	Pictorial	Abstract	Additional notes / guidance
317 + 46 = 363  If there are more than 10 in a column, then 10 need to be exchanged for 1 in the column above (e.g. 10 tens = 1 hundred), and then added in that column. Dienes / base 10 could still be used to help with understanding of place value	Children may also draw their own representations  Part-part-whole and bar models are also used to help understand the concept of addition – particularly to help visualise word problems.   	**3538** **+2758** **6296** Exchange using column addition is taught alongside concrete and pictorial strategies, and continual reminders of digit value is asserted – e.g. 5 hundreds + 7 hundreds = 12 hundreds 12 hundreds = 1 thousand + 2 hundreds	❖ Write column headings. ❖ Write the number of ones, tens, hundreds, and thousands in the correct column. ❖ Starting from the ones column add the numbers in each column exchanging from the next column if you need to.

YEAR 5 /6 - ADD NUMBERS WITH MORE THAN 4 DIGITS – COLUMN METHOD

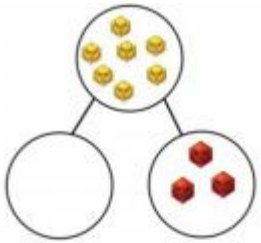
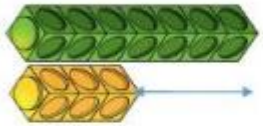
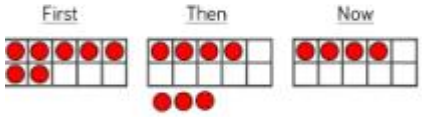
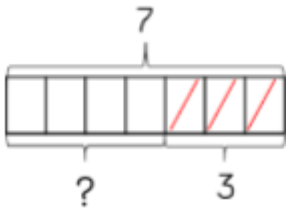
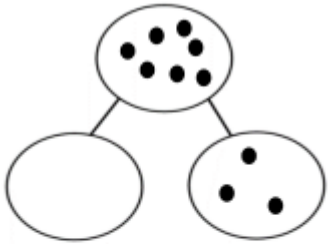
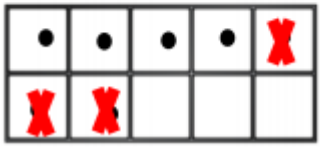
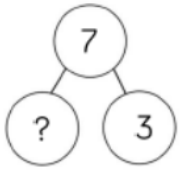
Concrete	Pictorial	Abstract	Additional notes / guidance
		 Without exchange 1 360 425 + 7 478 219 8 838 644 1 With exchange / renaming Exchange using column addition is taught alongside concrete and pictorial strategies, and continual reminders of digit value is asserted.	❖ Write column headings. ❖ Write the number of each in the correct column. ❖ Starting from the ones column. Add the numbers in each column carrying / renaming to the next column if you need to.

YEAR 5 /6 - ADD DECIMAL NUMBERS – COLUMN METHOD

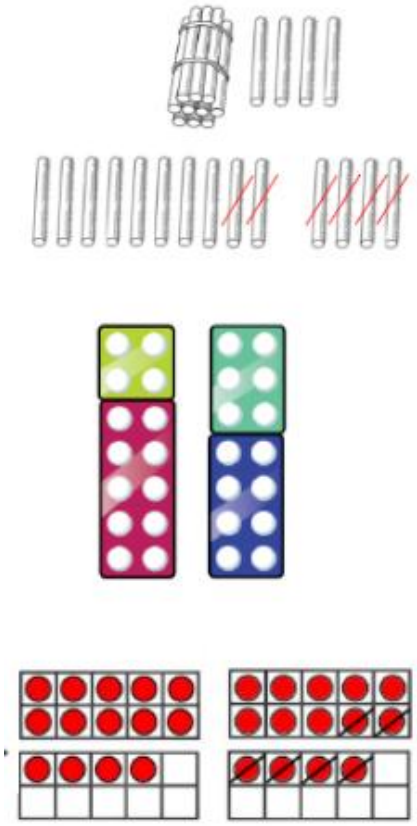
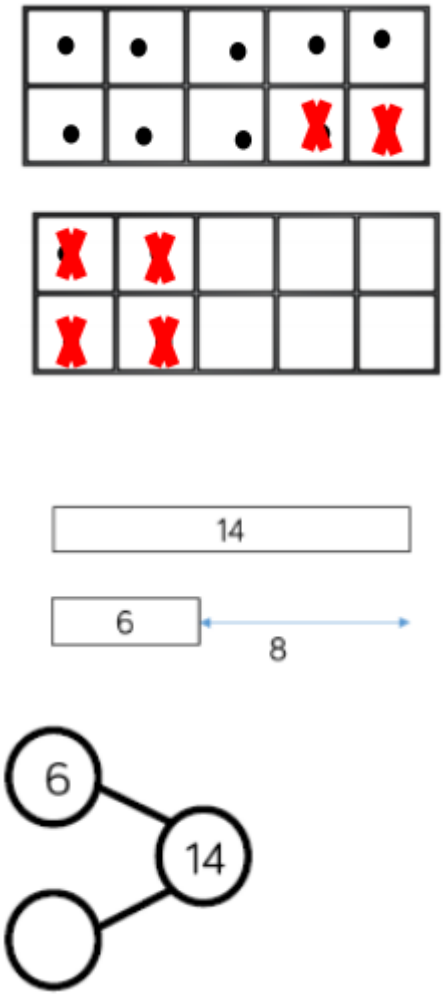
Concrete	Pictorial	Abstract	Additional notes / guidance
 		$3.65 + 2.41 = 6.06$ $\begin{array}{r} 3.65 \\ +\ 2.41 \\ \hline 6.06 \\ \hline 1 \end{array}$ Exchange using column addition is taught alongside concrete and pictorial strategies, and continual reminders of digit value is asserted – e.g. 5 tenths + 7 tenths = 12 tenths 12 tenths = 1 one + 2 tenths	❖ Write column headings. ❖ Write the number of each in the correct column. ❖ Starting from the furthest column to the right. Add the numbers in each column carrying / renaming to the next column if you need to.

Subtraction

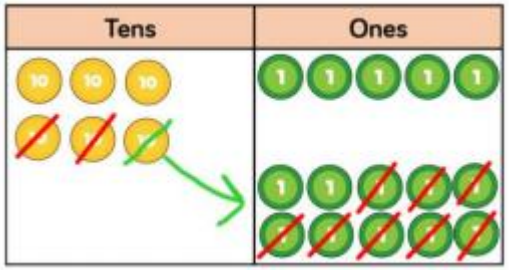
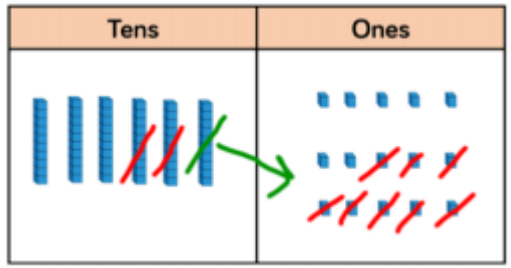
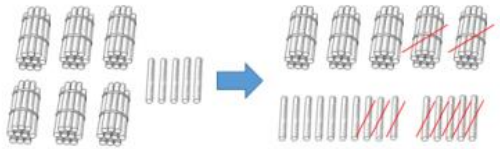
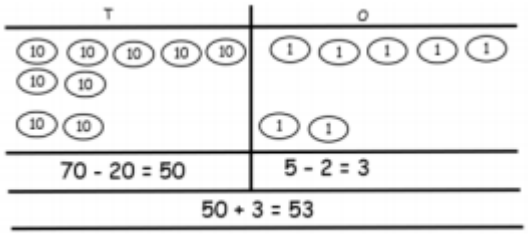
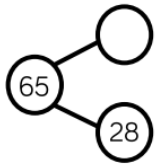
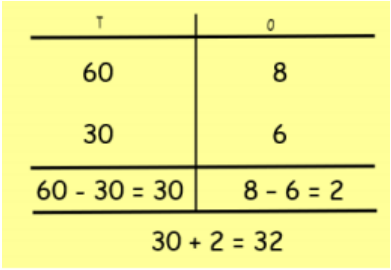
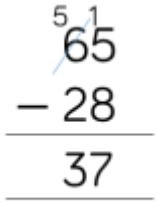
EYFS – SUBTRACTION WITHIN 10

Concrete	Pictorial	Abstract	Additional notes / guidance
    	   Moving to  	**7-3 =**	❖ Circle the first number on a number line. ❖ Make the number of jumps in the second number under the number line. ❖ Circle the answer.

YEAR 1 – SUBTRACTION WITHIN 10

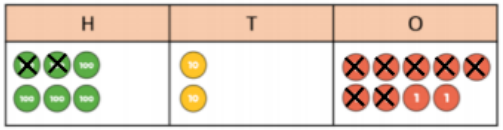
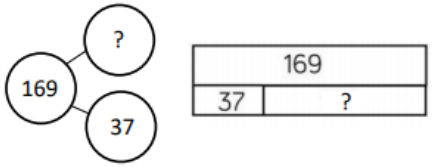
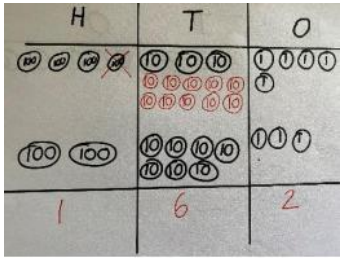
Concrete	Pictorial	Abstract	Additional notes / guidance
		 **14 – 6 = 8**	❖ Circle the first number on a number line. ❖ Make the number of jumps in the second number under the number line. ❖ Circle the answer.

YEAR 2 - SUBTRACTION OF TWO 2-DIGIT NUMBERS WITHIN 100

Concrete	Pictorial	Abstract	Additional notes / guidance
  	 75 - 22 	 68 – 36 	❖ Place the ones into the place value chart. ❖ Place the tens in the place value chart. ❖ Subtract the ones. ❖ Take / exchange a ten from the tens column if you need to. ❖ Subtract the tens. ❖ Subtract the ones.

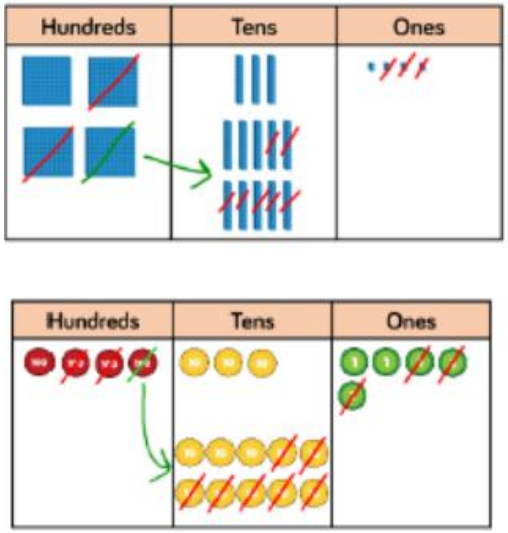
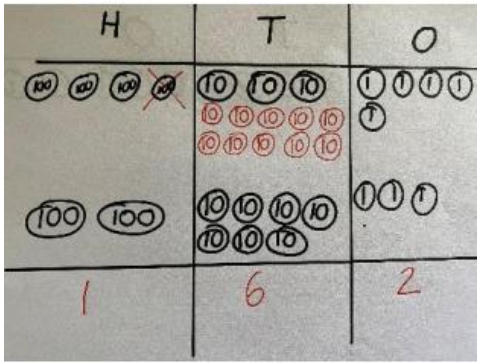
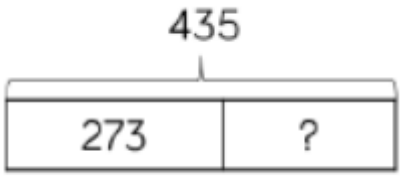
YEAR 3 – SUBTRACT NUMBERS WITH UP TO 3 DIGITS (without exchanging)

TO – O TO – TO HTO – TO HTO – HTO

Concrete	Pictorial	Abstract	Additional notes / guidance
352 – 122 = 230  **629 – 207 = 422**  Children look at each column (starting with the ones) and count how many counters/dienes are left in each.	  Children to draw their own representations.	**51** _ **568** = **517** Abstract column subtraction is taught alongside concrete and pictorial strategies to help link the concepts of column value - e.g. 6 tens - 5 tens = 1 ten	❖ Write column headings. ❖ Write the number of ones, tens, and hundreds in the correct column. ❖ Starting from the ones column add the numbers in each column exchanging from the next column if you need to.

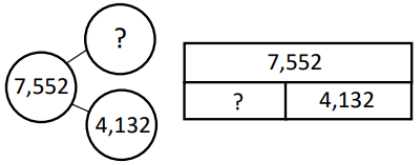
YEAR 3 - SUBTRACT NUMBERS WITH UP TO 3 DIGITS (with exchanging)

TO - O TO – TO HTO – O HTO – TO HTO - HTO

Concrete	Pictorial	Abstract	Additional notes / guidance
	 Children to draw their own representations 	$\begin{array}{r} {}^{3}\not{4}\,{}^{1}\not{3}5 \\ -\ 273 \\ \hline 262 \end{array}$ Exchange using column addition is taught alongside concrete and pictorial strategies, and continual reminders of digit value is asserted - e.g. 3 tens + 9 tens = 12 tens 12 tens = 1 hundred + 2 tens	❖ Write column headings. ❖ Write the number of ones, tens, and hundreds in the correct column. ❖ Starting from the ones column subtract the numbers in each column exchanging from the next column if you need to.

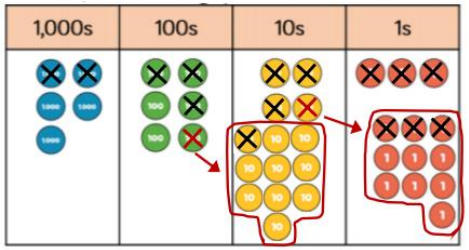
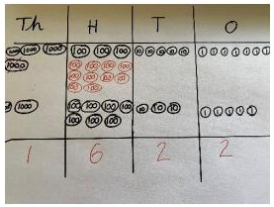
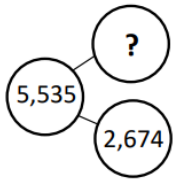
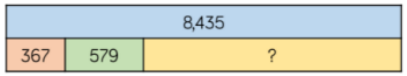
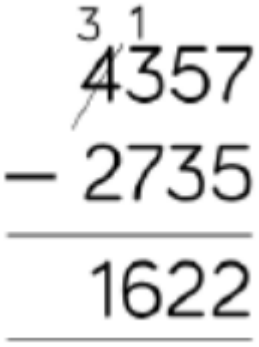
YEAR 4 - SUBTRACT NUMBERS WITH UP TO 4 DIGITS (without exchange)

up to ThHTO - ThHTO

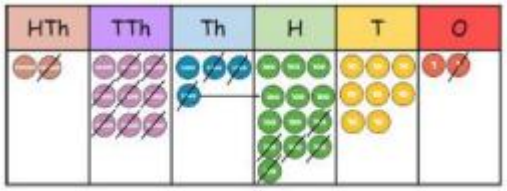
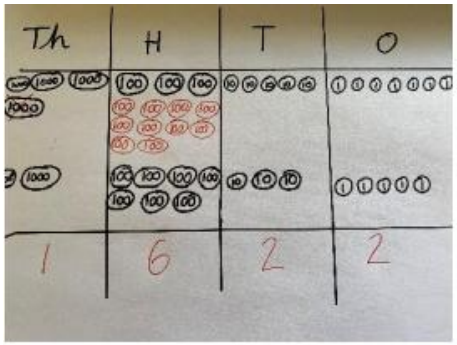
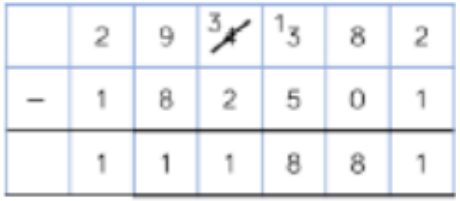
Concrete	Pictorial	Abstract	Additional notes / guidance
4,844 - 3,024 = 1,820  Children look at each column (starting with the ones) and count how many counters/dienes are left in each. Dienes could still be used to help with understanding of place value.	 	**3568** **-2341** **1227** Abstract column subtraction is taught alongside concrete and pictorial strategies to help link the concepts of column value – e.g. 6 tens - 4 tens = 2 tens	❖ Write column headings. ❖ Write the number of ones, tens, hundreds, and thousands in the correct column. ❖ Starting from the ones column subtract the numbers in each column exchanging / regrouping from the next column if you need to.

YEAR 4 – SUBTRACT NUMBERS WITH UP TO 4 DIGITS (with exchange)

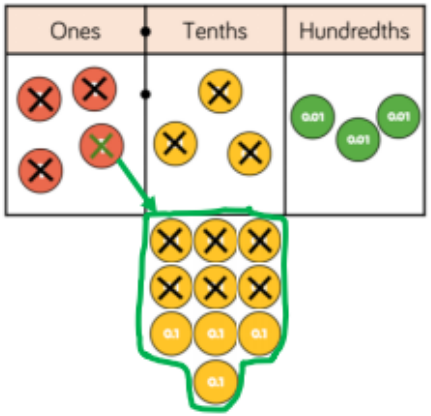
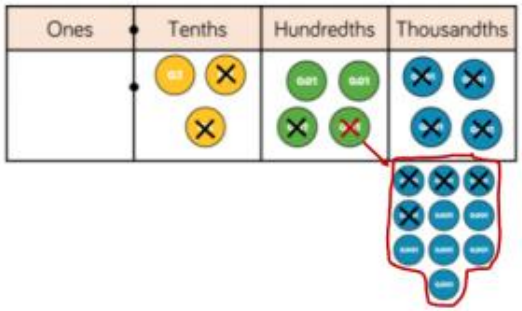
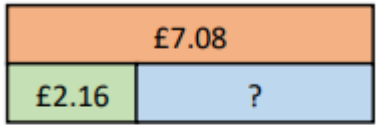
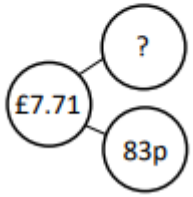
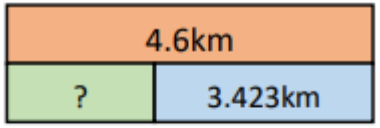
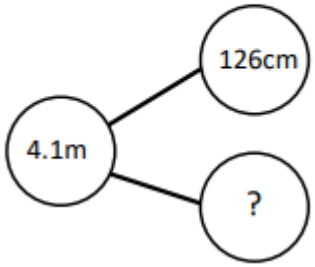
up to Th H T O + Th H T O

Concrete	Pictorial	Abstract	Additional notes / guidance
5,643 – 2,346 = 3,297 If we are subtracting more than we have from a column, then 1 from the column above needs to be exchanged for 10 (e.g. 1 ten = 10 ones), and then subtracted from.  	 Children to draw their own representations. Part-part-whole and bar models are also used to help understand the concept of subtraction – particularly to help visualise word problems.   	 Abstract column subtraction is taught alongside concrete and pictorial strategies to help link the concepts of column value –	❖ Write column headings. ❖ Write the number of ones, tens, hundreds, and thousands in the correct column. ❖ Starting from the ones column subtract the numbers in each column exchanging / regrouping from the next column if you need to.

YEARS- 5/6 SUBTRACT NUMBERS WITH MORE THAN 4 DIGITS

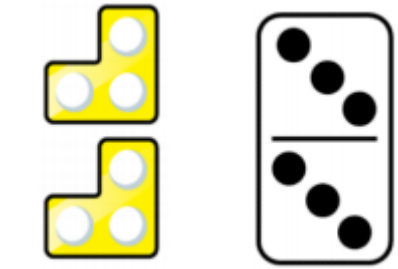
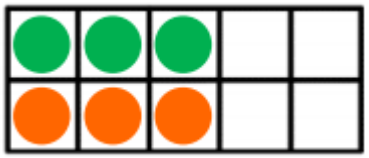
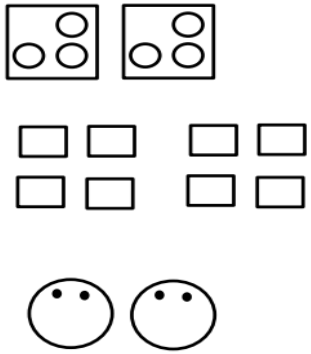
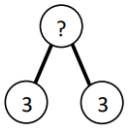
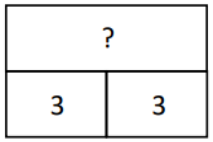
Concrete	Pictorial	Abstract	Additional notes / guidance
	 Children may draw their own representations.  730,529 244,945 ?	 Abstract column subtraction is taught alongside concrete and pictorial strategies to help link the concepts of column value –	❖ Write column headings. ❖ Write the number of ones, tens, hundreds, and thousands in the correct column. ❖ Starting from the ones column add the numbers in each column exchanging from the next column if you need to.

YEARS- 5/6 SUBTRACT DECIMALS INCLUDING MONEY AND MEASUREMENT

Concrete	Pictorial	Abstract	Additional notes / guidance
Year 5 E.g. 4·33 - 3·9 = 0·43  **Year 6** E.g. 0·344 – 0·218 = 0·126  	    	$\begin{array}{r} {}^{4}\not{3}.{}^{1}29 \\ -\ 2.57 \\ \hline 2.72 \end{array}$ $\begin{array}{r} {}^{2}\not{3}.{}^{1}{}^{1}{}^{1}\not{2}05 \\ -\ 2.572 \\ \hline 0.633 \end{array}$	If we are subtracting more than we have from a column, then 1 from the column above needs to be exchanged for 10 (e.g. 1 hundredth = 10 thousandths), and then subtracted from. Exchange using column subtraction is taught alongside concrete and pictorial strategies, and continual reminders of digit value is asserted - e.g. We exchange 1 one for 10 tenths. Our 10 tenths combine with our 1 tenth to make 11 tens. We can then take 5 tenths away from 11 tenths to be left with 6 tenths.

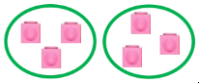
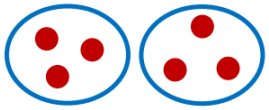
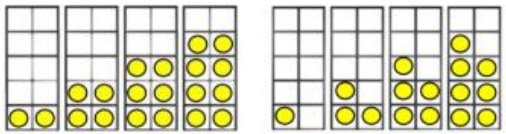
Multiplication

EYFS (FOCUS ON DOUBLING / ODD NUMBERS)

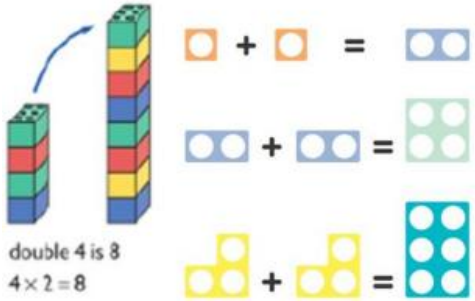
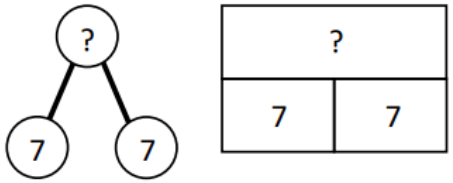
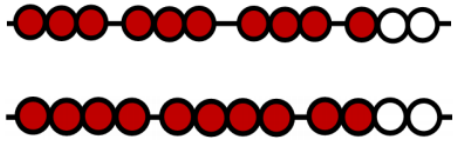
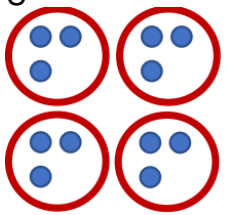
Concrete	Pictorial	Abstract	Additional notes / guidance
Doubling Children to recognise the patterns in doubled amounts. Children to use a wide range of manipulatives and equipment to show twice as much: e.g. Double 3 is…  	 As well as drawing pictures of representations showing doubling, children to use bar models and part-part-whole models: e.g. Double 3 is…   ? 3 3	Children to show doubling as repeated addition and solve mentally (using strategies such as counting on): e.g. 3 add 3 equals? Double 3 equals 6 Double 4 equals 8 Double 2 equals 4	Concepts to be consolidated through continuous provision as well as teacher and child led activities. Doubling using repeated addition in many different ways – using concrete and pictorial representations to solve problems**. No multiplication symbol to be used** and calculations should not be recorded formally.

EYFS (FOCUS ON DOUBLING / ODD NUMBERS)

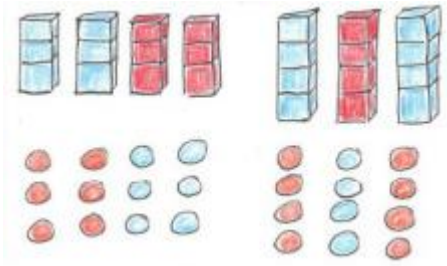
NB: In order to know odd/ even numbers the concept of halving will be introduced.

Concrete	Pictorial	Abstract	Additional notes / guidance
Halving by sharing into 2 groups. Children to recognising halving as sharing objects into 2 groups: e.g. What is half of 6? 	Children to share amount out into 2 groups by drawing: e.g. What is half of 6 	Children to use knowledge and understanding of grouping and sharing to begin to recall half of an amount: e.g. What is half of 6?	Concepts to be consolidated through continuous provision as well as teacher and child led activities.
Halving by finding groups of 2. Children to recognising halving as splitting objects into groups of 2: e.g. What is half of 6? 	Children to share amount out into groups of 2 by drawing: e.g. What is half of 6? 		
Odd / Even Numbers Children to recognise odd and even numbers by whether they can be halved. 	Children to draw quantities onto ten frames then decide if they are odd or even. 	Children to recall which numbers are odd and even: e.g. Is 3 odd or even?	

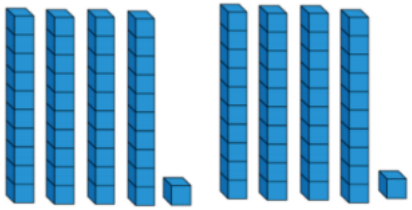
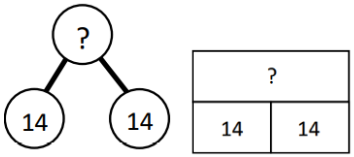
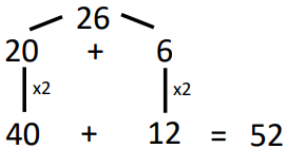
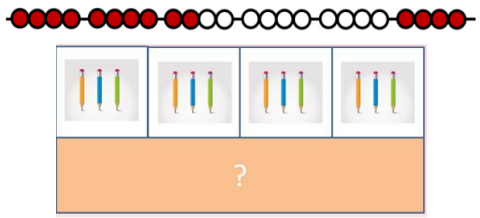
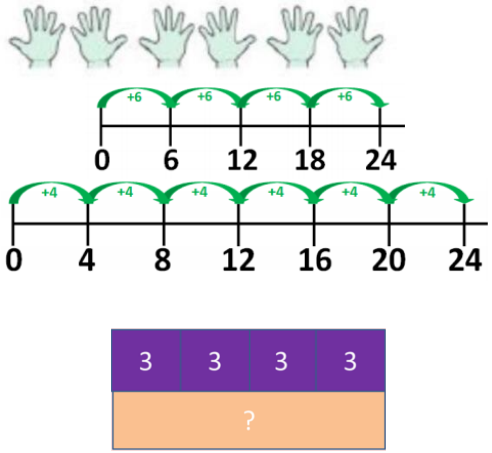
YEAR 1

Concrete	Pictorial	Abstract	Additional notes / guidance
Use practical activities using manipulatives including cubes and Numicon to demonstrate doubling. 	As well as drawing pictures of representations showing doubling, children to use bar models and part-part-whole models: e.g. Double 3 is... 	Children to show doubling as repeated addition and solve mentally (using strategies such as counting on): e.g. 7 lots of 2 = ? 7 + 7 = 14	
Repeated Addition Children to find groups of numbers using manipulatives: e.g. 4 lots of 3 	Children to draw pictures of groups of numbers: e.g. 4 lots of 3 = ? 	Children to use repeated addition to multiply numbers: e.g. 4 lots of 3 = ? 3 + 3 + 3 + 3 = 12 Write addition sentences to describe objects and pictures. 	

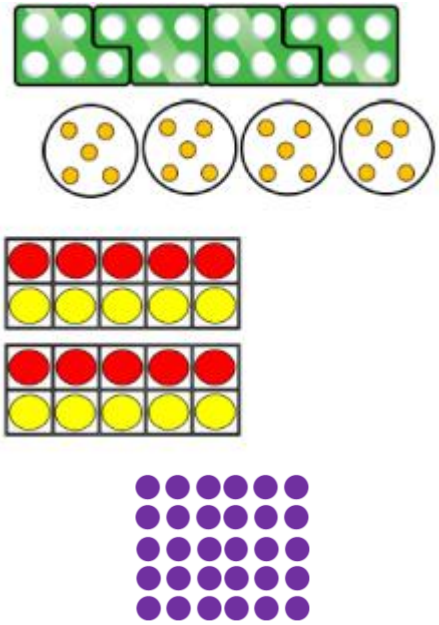
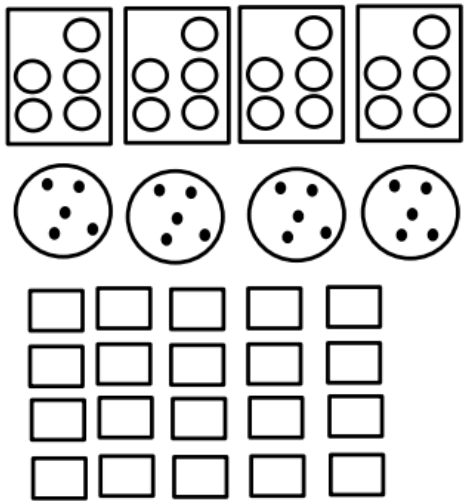
YEAR 1 – ARRAYS

Concrete	Pictorial	Abstract	Additional notes / guidance
Children to use objects (e.g. counters) to show an array of a multiplication: e.g. 3 x 5 = ? 	Children to draw arrays (e.g. with dots) to show multiplication: e.g. 6 x 2 = ?. 	3 x 2 = 6 2 x 5 = 10	

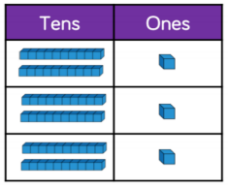
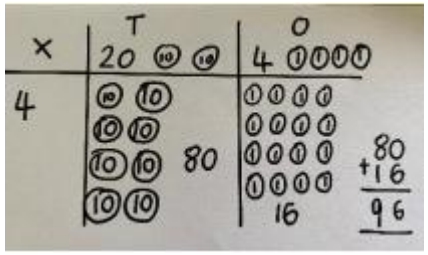
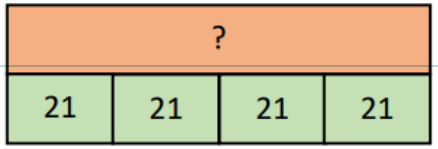
YEAR 2

Concrete	Pictorial	Abstract	Additional notes / guidance
Doubling (including numbers bigger than 10) 41 x 2 = ?  80 + 2 = 82	As well as drawing pictures of representations showing doubling, children to use bar models and part-part-whole models: e.g. Double 14 is … 	Children to partition number and then double: e.g. 26 x 2 = 	
Multiply numbers by using repeated addition Children to find groups of numbers using manipulatives: e.g. 6 x 4 = ? 	Number lines, counting sticks and bar models should be used to show representation of counting in multiples. 	Count in multiples of a number aloud. Write sequences with multiples of numbers. 0, 2, 4, 6, 8, 10 0, 3, 6, 9, 12, 15 0, 5, 10, 15, 20, 25, 30 Children to use repeated addition to multiply numbers: e.g. 4 x 6 = 6 + 6 + 6 + 6 = 24 6 x 4 = 4 + 4 + 4 + 4 + 4 + 4 = 24	

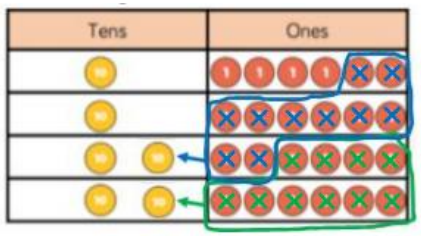
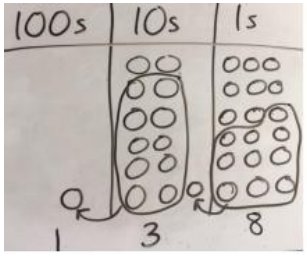
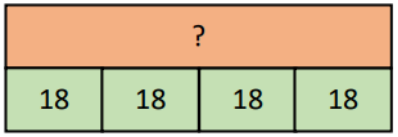
YEAR 2

Concrete	Pictorial	Abstract	Additional notes / guidance
Using arrays to show multiplication. Children to use objects (e.g. counters) to show an array of a multiplication: e.g. 6 x 5 = ? 		$5+5+5+5=20$ $4 \times 5 = 20$ $5 \times 4 = 20$ One bag holds 5 apples. How many apples do 4 bags hold?	

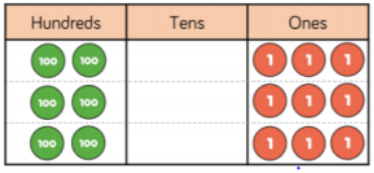
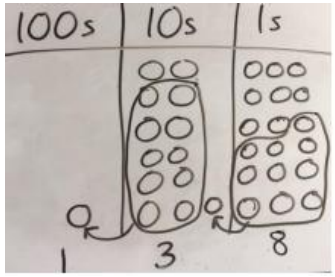
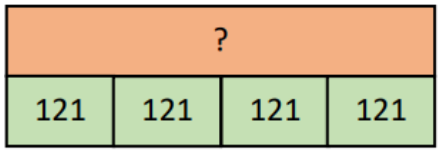
YEAR 3

Concrete	Pictorial	Abstract	Additional notes / guidance
Multiply (without exchanging) TO x O E.g. 21 x 3 = 63  34 x 2 = 68  After multiplying, children look at each column (starting 9 tens with the ones) and count how many counters/dienes are in each.	 Bar models are also used to help understand the concept of multiplication – particularly to help visualise word problems.  	**21 X 3** Also set out formally 21 X 3	Abstract column multiplication is taught alongside concrete and pictorial strategies to help link the concepts of column value – e.g. 3 tens x 3 = 9 tens

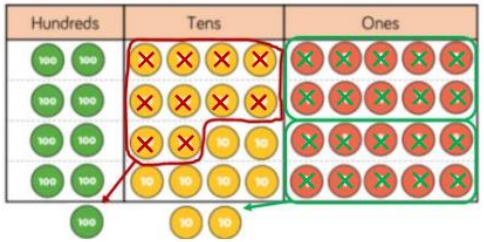
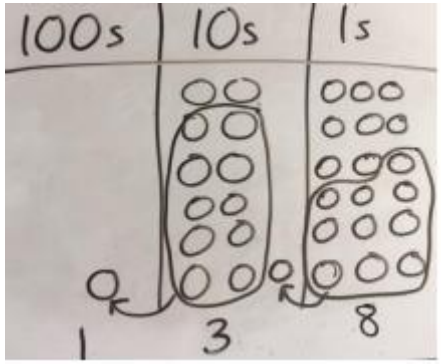
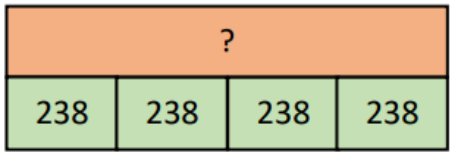
YEAR 3

Concrete	Pictorial	Abstract	Additional notes / guidance
Multiply (with exchanging) TO x O 16 x 4 = 64  If there are more than 10 in a column, then each 10 needs to be exchanged for 1 in the column above (e.g. 10 ones = 1 ten), and then added in that column.	 Example Drawings / pictures of Dienes and place value counters used as opposed to physical equipment. Bar models are also used to help understand the concept of multiplication – particularly to help visualise word problems.  	46 x 4 184 1 2 Exchange using column multiplication is taught alongside concrete and pictorial strategies, and continual reminders of digit value is asserted – e.g. 4 tens x 4 = 16 tens = 1 hundred + 6 tens	Expanded column multiplication could be used if exchanging is confused. 46 x 4 24 (6x4) 160 (40x4) 184

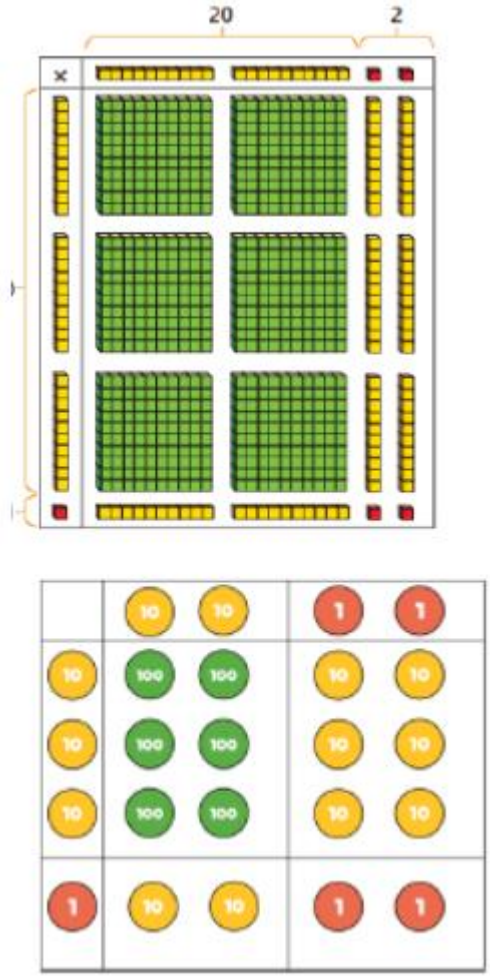
YEAR 4

Concrete	Pictorial	Abstract	Additional notes / guidance
Multiply (without exchanging) HTO x O 203 x 3 = 609  After multiplying, children look at each column (starting with the ones) and count how many counters/dienes are in each. Dienes / Base ten equipment should still be used to help with understanding of place value.	 Example Pictures / drawings of Dienes /Base ten and place value counters used as opposed to physical equipment. Bar models are also used to help understand the concept of multiplication – particularly to help visualise word problems.  	302 x 3 906	

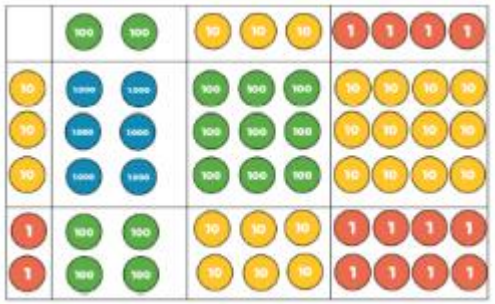
YEAR 4

Concrete	Pictorial	Abstract	Additional notes / guidance
Multiply (with exchanging) HTO x O 245 x 4 = 980  If there are more than 10 in a column, then each 10 needs to be exchanged for 1 in the column above (e.g. 20 ones = 2 tens), and then added in that column. Dienes/Base ten should still be used to help with the understanding of place value.	 Example Pictures of Dienes and place value counters used as opposed to physical equipment. Bar models are also used to help understand the concept of multiplication – particularly to help visualise word problems.  	546 x 4 2184 2 1 2	

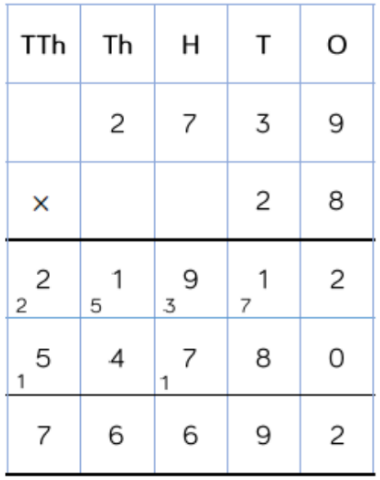
YEARS 5 (When children start to multiply 2d x 2d 3d × 3d and 4d × 2d etc., they should be confident with the abstract)

Concrete	Pictorial	Abstract	Additional notes / guidance
Two digits by two digits 31 x 22  	<table><tr><td>×</td><td>20</td><td>2</td></tr><tr><td>30</td><td>600</td><td>60</td></tr><tr><td>1</td><td>20</td><td>2</td></tr></table>	<table><tr><td></td><td>H</td><td>T</td><td>O</td></tr><tr><td></td><td></td><td>2</td><td>2</td></tr><tr><td>×</td><td></td><td>3</td><td>1</td></tr><tr><td></td><td></td><td>2</td><td>2</td></tr><tr><td></td><td>6</td><td>6</td><td>0</td></tr><tr><td></td><td>6</td><td>8</td><td>2</td></tr></table>	❖ Multiply all the numbers on the top row by the ones digit. ❖ Remember to add ‘the zero for a place holder’ ❖ Multiply all the numbers on the top row by the tens digit. ❖ Add the two lines together.

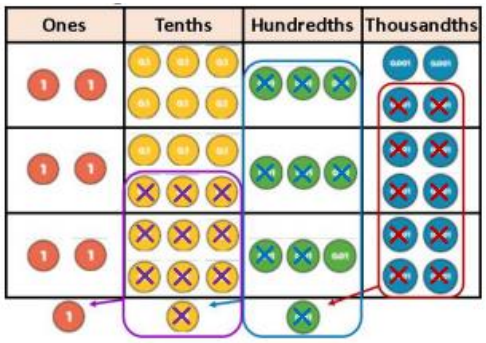
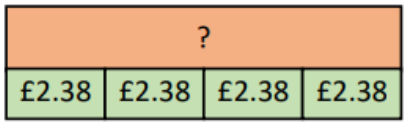
YEAR 5

Concrete	Pictorial	Abstract	Additional notes / guidance
3 digits by 2-digit number 234 x 32 	<table><tr><td>×</td><td>200</td><td>30</td><td>4</td></tr><tr><td>30</td><td>6,000</td><td>900</td><td>120</td></tr><tr><td>2</td><td>400</td><td>60</td><td>8</td></tr></table>	<table><tr><td>Th</td><td>H</td><td>T</td><td>O</td></tr><tr><td></td><td>2</td><td>3</td><td>4</td></tr><tr><td>×</td><td></td><td>3</td><td>2</td></tr><tr><td></td><td>4</td><td>6</td><td>8</td></tr><tr><td>1 7</td><td>1 0</td><td>2</td><td>0</td></tr><tr><td>7</td><td>4</td><td>8</td><td>8</td></tr></table>	❖ Multiply all the numbers on the top row by the ones digit. ❖ Remember to add ‘the zero for a place holder’ ❖ Multiply all the numbers on the top row by the tens digit. ❖ Add the two lines together.

YEAR 6

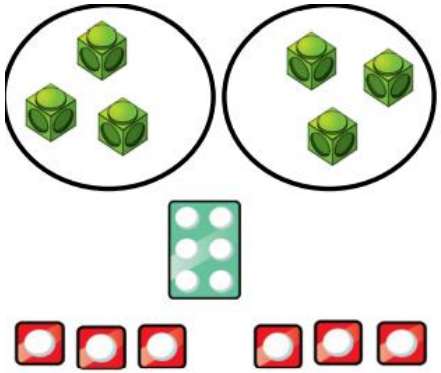
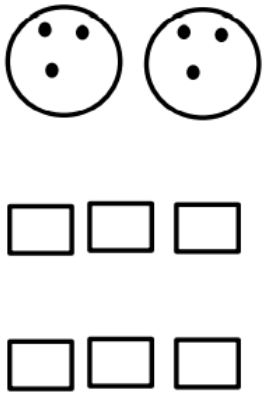
Concrete	Pictorial	Abstract	Additional notes / guidance
			❖ Multiply all the numbers on the top row by the ones digit. ❖ Remember to add 'the zero for a place holder' ❖ Multiply all the numbers on the top row by the tens digit. ❖ Add the two lines together

YEARS 5 / 6 MULTIPLYING DECIMALS

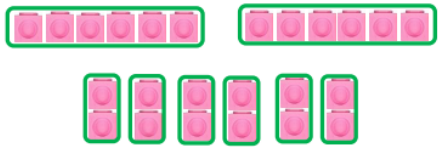
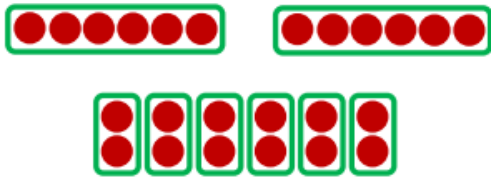
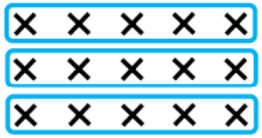
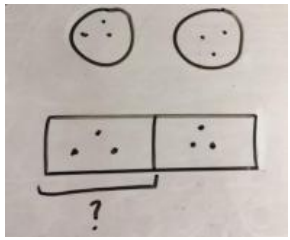
Concrete	Pictorial	Abstract	Additional notes / guidance
Multiply decimals (up to 3 places) by 1-digit numbers (including money) 2.634 x 3 = 7.902  	Pictures of place value counters used as opposed to physical equipment. Bar models are also used to help understand the concept of multiplication – particularly to help visualise word problems.  	5.406 x 4 2 1 2 21.624	Exchange using column multiplication is taught alongside concrete and pictorial strategies, and continual reminders of digit value is asserted – e.g. 6 thousandths x 4 = 24 thousandths = 2 hundredths + 4 thousandths

Division

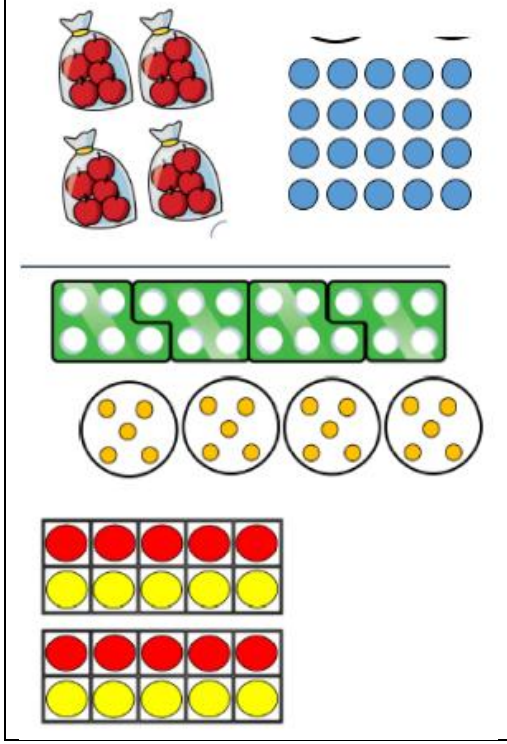
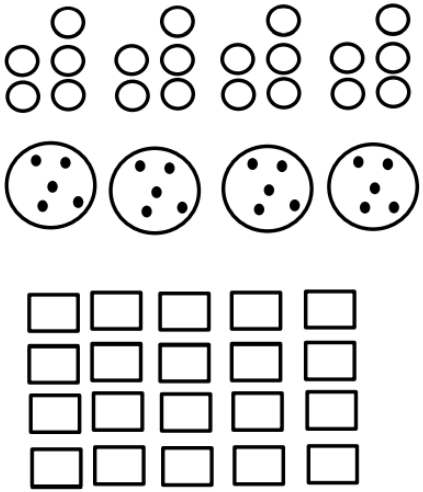
EYFS (See odd and even numbers in multiplication section)

Concrete	Pictorial	Abstract	Additional notes / guidance
		Half of 6 equals 3	Division by sharing amounts into equal groups to half – using concrete and pictorial representations to solve problems. No division symbol to be used and calculations should not be recorded formally.

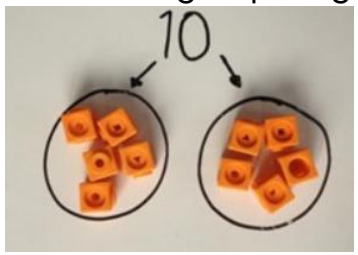
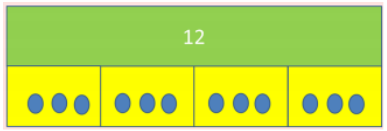
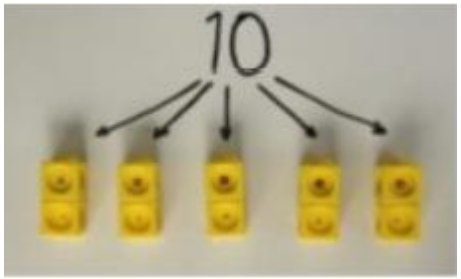
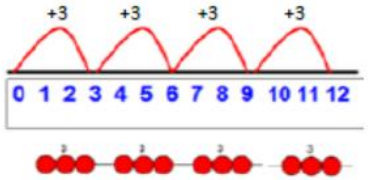
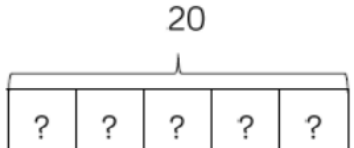
YEAR 1

Concrete	Pictorial	Abstract	Additional notes / guidance
Halving by sharing or splitting Children to recognising halving as sharing or splitting: e.g. What is half of 12? 	Children to share/split amounts in half by drawing: e.g. What is half of 12?	Children to use knowledge of halving and doubling to recall half of an amount: e.g. What is half of 12? 	
Divide numbers by sharing. Children to share a number of objects into a certain number of groups: e.g.15 shared into 3 groups 	Children to share amounts into a certain number of groups by drawing: e.g. 15 shared into 3 groups  	Children to use knowledge of grouping, sharing and arrays to recall division facts: e.g. 15 shared into 3 groups	Division sign does not need to be introduced.

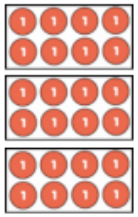
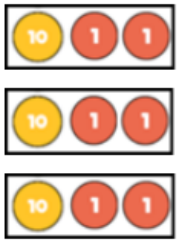
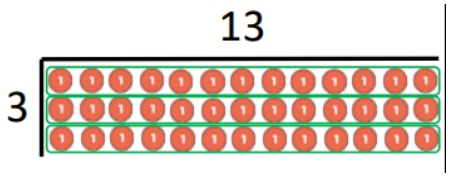
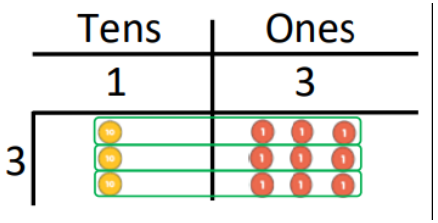
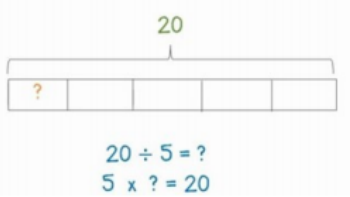
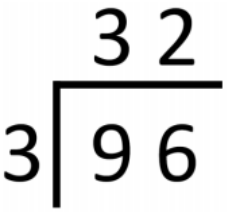
YEAR 1

Concrete	Pictorial	Abstract	Additional notes / guidance
Division by grouping 		How many groups of 4 are there in 20? There are 4 groups of 5 in 20. There are 20 apples altogether. They are put in bags of 5. How many bags are there?	Division by grouping and counting the number of groups. Grouping encourages children to count in multiples and links to repeated subtraction. They can use concrete representations in fixed groups such as number shapes which helps to show the link between multiplication and division. No division symbol to be used.

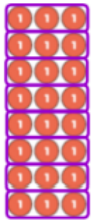
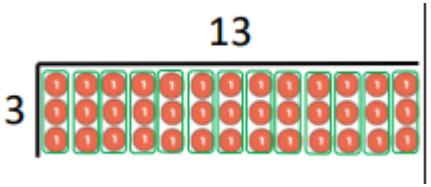
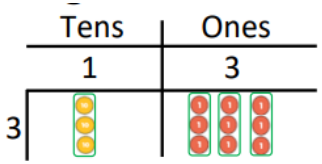
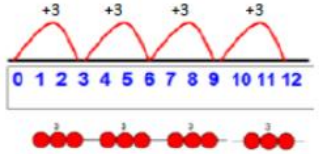
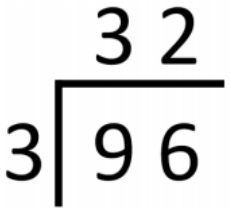
YEAR 2

Concrete	Pictorial	Abstract	Additional notes / guidance
Children to share a number of objects into a certain number of groups: e.g.  I have 10 cubes; can you share them equally in 2 groups?	Share 8 flowers into 2 groups  8 ÷ 2 = 4  12 ÷ 4 = 3	12 ÷ 3 = 4	Division by sharing amounts into equal groups – using concrete and pictorial representations to solve problems. Division symbol to be used.
Division as grouping Divide quantities into equal groups. Use cubes, counters, objects, or place value counters to aid understanding.  	Use number lines for grouping.  12 ÷ 3 = 4 Use Bar models  	28 ÷ 7 = 4 Divide 28 into 7 groups. How many are in each group?	Think of the bar as a whole. Split it into the number of groups you are dividing by and work out how many would be within each group

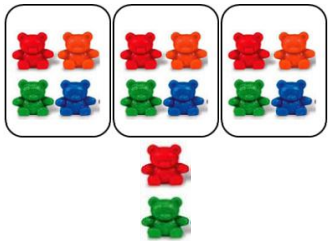
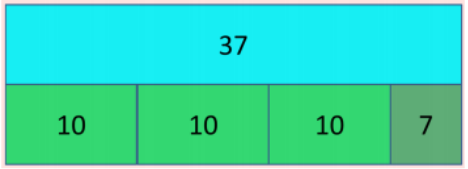
YEAR 3

Concrete	Pictorial	Abstract	Additional notes / guidance
Dividing by sharing TO ÷ O 24 ÷ 3 = 8 24 ones are shared between 3 groups.  36 ÷ 3 = 12 3 tens and 6 ones are shared between 3 groups. 	39 ÷ 3 = 13  39 shared between 3 groups.  3 tens and 9 ones shared between 3 groups. Continue to use the Bar model 	 Sarah has 30 chocolates that she wants to share equally between herself and four friends. How many will they have each?	Formal written method of division is taught alongside concrete and pictorial representations. Division can be thought of as 'sharing' (i.e. if I share 9 tens between 3 groups, how many would be in each group?) or 'grouping' (i.e. how many groups of three can I take out of 9?)

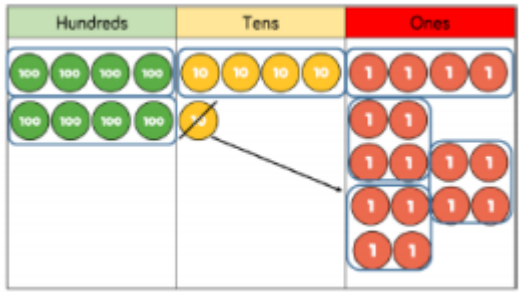
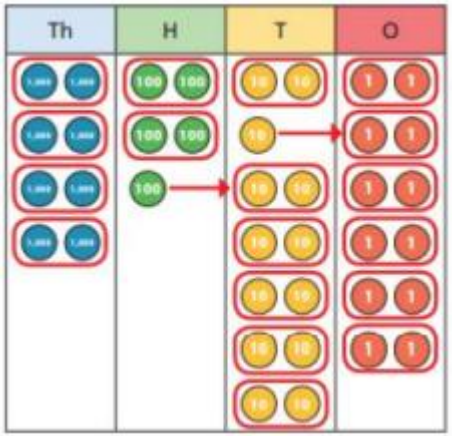
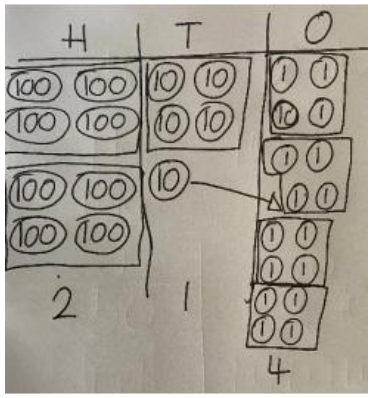
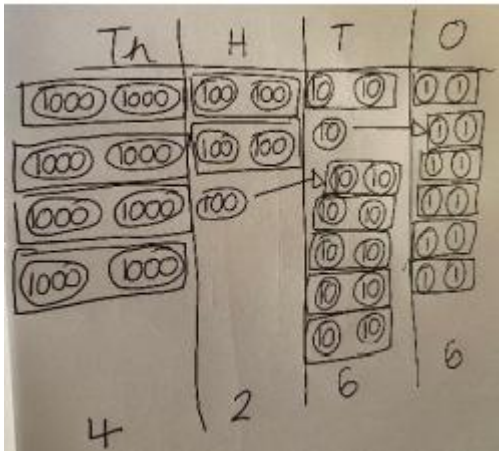
YEAR 3

Concrete	Pictorial	Abstract	Additional notes / guidance
Dividing by grouping TO ÷ O 24 ÷ 3 = 8 How many 3s can be taken from 24 ones?  36 ÷ 3 = 12 How many 3s can be taken from 3 tens? How many 3s can be taken from 6 ones? 	39 ÷ 3 = 13  How many groups of 3 can you take from 39?  How many groups of 3 can you take from 3 tens? How many groups of 3 can you take from 9 ones? Number lines may also be used for grouping at year 3.  	 How many groups of 6 in 24? 24 ÷ 6 = 4	Formal written method of division is taught alongside concrete and pictorial representations. Division can be thought of as 'sharing' (i.e. if I share 9 tens between 3 groups, how many would be in each group?) or 'grouping' (i.e. how many groups of three can I take out of 9?)

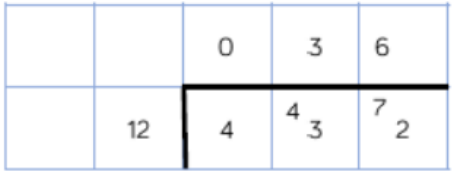
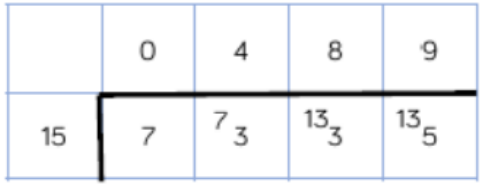
YEAR 3

Concrete	Pictorial	Abstract	Additional notes / guidance
Division with remainders. 14 ÷ 3 = Divide objects between groups and see how much is left over. 	Jump forward in equal jumps on a number line then see how many more you need to jump to find a remainder.  Draw dots and group them to divide an amount and clearly show a remainder.  Use bar models to show division with remainders  	Complete written divisions and show the remainder using r.  	

YEARs 4-6

Concrete	Pictorial	Abstract	Additional notes / guidance
Divide 3 and 4 digits by 1 digit – grouping with remainders   	  	  	❖ Start with the largest place value and divide by divisor. ❖ Move through each of the place value and divide by the divisor. ❖ Calculate any remainders.

YEARS 4-6

Concrete	Pictorial	Abstract	Additional notes / guidance
	<table><tr><td colspan="2">12</td></tr><tr><td>2 x</td><td>24</td></tr><tr><td>3 x</td><td>36</td></tr><tr><td>4 x</td><td>48</td></tr><tr><td>5 x</td><td>60</td></tr><tr><td>6 x</td><td>72</td></tr><tr><td>7 x</td><td>84</td></tr><tr><td>8 x</td><td>108</td></tr><tr><td>9 x</td><td>132</td></tr><tr><td>10 x</td><td>144</td></tr></table>	 	❖ Create a toolbox for multiples of the number we are dividing by. ❖ Divide each digit by the number we are dividing by. ❖ Calculate any remainders.

YEARS 4-6

Abstract	Additional notes / guidance
Long Division examples:	

432 ÷ 15 becomes

$$\begin{array}{r|rrr} & & 2 & 8 \quad r\,12 \\ \hline 15 & 4 & 3 & 2 \\ & 3 & 0 & 0 \\ \hline & 1 & 3 & 2 \\ & 1 & 2 & 0 \\ \hline & & 1 & 2 \end{array}$$

Answer: 28 remainder 12

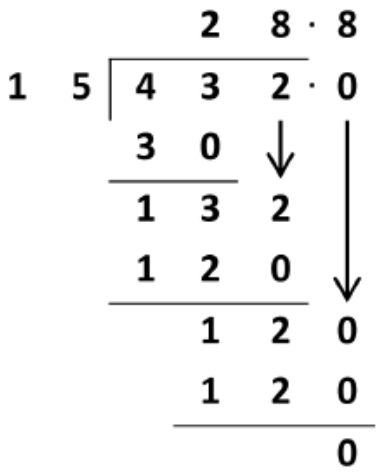
432 ÷ 15 becomes

$$\begin{array}{r|rrrl} & & 2 & 8 & \\ \hline 15 & 4 & 3 & 2 & \\ & 3 & 0 & 0 & 15\times20 \\ \hline & 1 & 3 & 2 & \\ & 1 & 2 & 0 & 15\times8 \\ \hline & & 1 & 2 & \end{array}$$

$$\frac{\cancel{12}}{\cancel{15}} = \frac{4}{5}$$

Answer: $28\frac{4}{5}$

YEARS 4-6

Abstract	Additional notes / guidance
Long Division examples: 432 ÷ 15 becomes  Answer: 28·8	

YEARS 4-6

Abstract	Additional notes / guidance
Long Division examples:	

Dividing using long division

1.) There are **0** 27s in 9.	$\begin{array}{r} 0 \\ 27\overline{)9936} \end{array}$	2.) There are 3 27s in 99. 3 x 27 = 81	$\begin{array}{r} 03 \\ 27\overline{)9936} \end{array}$
3.) 99 – 81 = 18 left over	$\begin{array}{r} 03 \\ 27\overline{)9936} \\ \underline{81} \\ 18 \end{array}$	4.) 18 hundreds joins with 3 tens to make 183 tens.	$\begin{array}{r} 03 \\ 27\overline{)9936} \\ \underline{81\downarrow} \\ 183 \end{array}$
5.) There are **6** 27s in 183. 6 x 27 = 162	$\begin{array}{r} 036 \\ 27\overline{)9936} \\ \underline{81\downarrow} \\ 183 \end{array}$	6.) 183 – 162 = 21 left over	$\begin{array}{r} 036 \\ 27\overline{)9936} \\ \underline{81\downarrow} \\ 183 \\ \underline{162} \\ 021 \end{array}$
7.) The **21** tens are exchanged for 210 ones to make 216 ones.	$\begin{array}{r} 036 \\ 27\overline{)9936} \\ \underline{81\downarrow} \\ 183 \\ \underline{162} \\ 0216 \end{array}$	5.) There are **8** 27s in 216. with 0 left over	$\begin{array}{r} 0368 \\ 27\overline{)9936} \\ \underline{81\downarrow} \\ 183 \\ \underline{162} \\ 0216 \end{array}$